For Reference

NOT TO BE TAKEN FROM THIS ROOM

For Reference

NOT TO BE TAKEN FROM THIS ROOM

Ex libris
Universitatis
Albertaensis

UNIVERSITY OF ALBERTA
QUAECUMQUE VERA

University of Alberta
Printing Department

Digitized by the Internet Archive
in 2019 with funding from
University of Alberta Libraries

https://archive.org/details/componentcurvean00thom

THE UNIVERSITY OF ALBERTA

[illegible] CURVE ANALYSIS OF [illegible]

by

[illegible]

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF [illegible]

EDMONTON, ALBERTA

[illegible]

Thesis
1964(F
#41

THE UNIVERSITY OF ALBERTA

COMPONENT CURVE ANALYSIS OF CONCEPT ATTAINMENT

by

THOMAS OWENS MAGUIRE

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF EDUCATION

DEPARTMENT OF EDUCATIONAL PSYCHOLOGY

EDMONTON, ALBERTA
AUGUST,1964

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Component Curve Analysis of Concept Attainment" submitted by Thomas Owens Maguire in partial fulfillment of the requirements for the degree of Master of Education.

ABSTRACT

This dissertation was designed to investigate the relationship between personality and cognitive variables and concept acquisition as analyzed by the method of component curves. A further purpose was to describe the behaviour of the component curves under conditions of transfer and under conditions of changing difficulty.

Concept formation tasks were devised that produced sequences of scores which were considered to be representative of the learning processes. One group of subjects was given two tasks of equal difficulty and another group of subjects was given two tasks of different difficulty.

The matrices of scores were broken down into component curves and individual loadings. The canonical correlation between the individual loadings of group one task one and the personality and cognitive variables was found to be significant at only the .20 level. The important psychological variables seemed to be serious versus enthusiastic, uncontrolled versus controlled, and reasoning.

The group was divided into sub-groups according to which curve they loaded most heavily. A discriminant function was calculated using the personality and cognitive

variables. Again the three aforementioned variables seemed to be most important. The memory test was found to be of some importance also.

Using two equally difficult tasks successively on the same group it was found that the curves showed earlier acceleration on the second than on the first. This was an indication that transfer had occurred. This was verified using a descriptive technique.

The investigation of task difficulty and curve complexity was unresolved inasmuch as the increase in difficulty was too great for the subjects.

ACKNOWLEDGEMENT

The writer wishes to acknowledge a considerable debt to the many people whose co-operation and assistance made possible the completion of this thesis.

In particular, thanks are due to Dr. S. M. Hunka who first introduced the writer to the idea of component curve analysis and who offered helpful criticism at every turn. Thanks are also due to the other members of the committee, Dr. L. D. Nelson and Dr. C. M. Christensen for their suggestions.

The writer is also indebted to other members of the department staff, particularly Dr. R. J. C. Harper for several helpful suggestions.

Thanks is expressed to the University of Alberta for providing financial assistance that enabled the writer to pursue his graduate studies.

Finally, appreciation is expressed to the principal, staff, and students of Westglen Junior High School, Edmonton who participated in the pilot study and to Sister Marie Leo, the staff, and students of Mount Carmel School, Edmonton who participated in the study itself.

TABLE OF CONTENTS

CHAPTER PAGE

I. INTRODUCTION 1

II. INTRODUCTION TO THE REVIEW 3

Justification for Classifying Concept Formation 10

Cognitive Variables Affecting Concept Formation 12

Personality Variables and Problem Solving 15

Task Difficulty and Concept Formation . . 16

Transfer and Concept Formation 17

III. DEFINITIONS AND HYPOTHESES 19

Definitions 19

Concept Formation 19

Difficulty 19

Complexity of the Learning Function . . . 19

Transfer 19

Hypotheses 20

IV. EXPERIMENTAL DESIGN 21

The Sample 21

The Task 21

Tests Used 23

Holzinger-Crowder Uni-Factor Tests (1958) Form Am 23

CHAPTER PAGE

Junior-Senior High School Personality Questionnaire (HSPQ) 23

Memory Task 24

Experimental Procedure 26

Task Presentation 26

Analysis 27

Rotation 28

Test Presentation and Analysis 28

V. THE RESULTS 30

Selected Description of Performance on Curve One 49

Selected Description of Performance on Curve Two 50

The Relationship Between Groups, Personality Variables and Cognitive Variables 52

Results: Transfer 61

Results: Task Difficulty 67

VI. SUMMARY OF FINDINGS AND IMPLICATIONS FOR FUTURE RESEARCH 70

Results 70

CHAPTER PAGE

Implications For Further Research 71

General Implications 71

Implications with Respect to the Use of the Technique 72

BIBLIOGRAPHY 74

APPENDIX A 80

APPENDIX B 83

APPENDIX C 88

APPENDIX D 91

APPENDIX E 95

APPENDIX F 108

LIST OF TABLES

TABLE PAGE

I. Titles and Designations of the Fourteen Personality Dimensions 25

II. Calculation of the Runs Test for Determination of the Number of Significant Curves 32

III. Unrotated Individual Loadings, Group B Problem One 39

IV. Unrotated Individual Loadings, Group B Problem Two 42

V. Individual Loadings on Rotated Reference Curves, Group A, Problem One 46

VI. Individual Loadings on Rotated Reference Curves, Group A, Problem Two 48

VII. Intercorrelations Among Predictors and Individual Loadings, Group A, Problem One 54

VIII. Intercorrelations Among Selected Predictors and Individual Loadings . . 58

IX. Variable Weights for Maximum Canonical Correlations Predictors 59

X. Table of Coefficients and Multiple Correlations of Each Step of the Discriminant Function 62

TABLE PAGE

XI. Means of Variables Selected for Discriminant Function 63

XII. Intercorrelation Between Individual Loadings on the Two Component Curves for Problems One and Two of Group A . 65

XIII. Number of Subjects in Group A Distributed According to Group Membership in Problem One and Problem Two 68

XIV. Score Matrix Group A, Problem One . . . 95

XV. Score Matrix Group A, Problem Two . . 96

XVI. Score Matrix Group B, Problem One . . . 97

XVII. Score Matrix Group B, Problem Two . . . 98

XVIII. First Five Eigenvalues and Eigenvectors Group A, Problem One 99

XIX. First Five Eigenvalues and Eigenvectors Group A, Problem Two 99

XX. First Five Eigenvalues and Eigenvectors Group A, Problem One 100

XXI. First Five Eigenvalues and Eigenvectors Group B, Problem Two 100

XXII. Unrotated Loadings for Component Curves Group A, Problem One 101

XXIII. Unrotated Loadings for Component Curves Group A, Problem Two 102

XXIV. Unrotated Loadings for Component Curves Group B, Problem One 103

TABLE PAGE
XXV. Unrotated Loadings for Component Curves Group B, Problem Two 104
XXVI. Unrotated Individual Loadings, Group A, Problem One 105
XXVII. Unrotated Individual Loadings, Group A, Problem Two 106
XXVIII. Variance-Covariance Matrix for Discriminant Function 108

LIST OF FIGURES

FIGURE PAGE

1. First Unrotated Component Curve, Group A, Problem One 33
2. Second Unrotated Component Curve, Group A, Problem One 33
3. Third Unrotated Component Curve, Group A, Problem One 34
4. First Unrotated Component Curve, Group A, Problem Two 35
5. Second Unrotated Component Curve, Group A, Problem Two 35
6. Third Unrotated Component Curve Group A, Problem Two 36
7. First Unrotated Component Curve Group B, Problem One 37
8. Second Unrotated Component Curve Group B, Problem One 38
9. First Unrotated Component Curve, Group B, Problem Two 40
10. Second Unrotated Component Curve, Group B, Problem Two 41

FIGURE PAGE

11. Interfactor Loadings Group A, Problem One 43
12. Interfactor Loadings Group A, Problem Two 44
13. First Rotated Reference Curve, Group A, Problem One 45
14. Second Rotated Reference Curve, Group A, Problem One 45
15. First Rotated Reference Curve, Group A, Problem Two 47
16. Second Rotated Reference Curve, Group A, Problem Two 47

CHAPTER I

INTRODUCTION

Despite the fact that the field of concept formation has undergone intensive research in the past few decades, relatively little is known about how individuals acquire new concepts. Even less is known about the characteristics of the learner.

Research in this area is divisable into two fields. The first area is the investigation of the end product. These studies investigate concept formation from the point of view of the developmental aspect. Such research is aimed at discovering at what stage particular concepts are acquired by an individual and determining the breadth of concepts. In this area, it is the concept that is studied not the formation.

The second area of study has been the process of formation itself. What does a person actually do to acquire a concept? This area of study has been more recent and up to the present time has been tedious, requiring the individual study of subjects in the process of acquiring concepts.

The present study is an attempt to adapt the method of component curve analysis to studying the process of concept formation. This in effect brings the time saving

methods of group analysis to bear on the study of concept acquisition.

Essentially the study is an effort to delineate learning parameters or component curves, and to describe individual behavior in terms of these parameters. The parameters themselves are to be studied to determine their relationship to certain cognitive and personality variables. In this way it is hoped that a fairly useful description of the parameters will be possible.

Because the method to be used in determining the learning parameters is new, the extent of its possibilities in educational and psychological research is as yet undiscovered. Therefore a second purpose of this study is to illustrate a further use of the technique in the above fields.

If the method of component curves is to become more widely used in psychology and in particular in the area of concept formation, further investigation of the characteristics of the curves is necessary. This then is the third purpose of the study.

CHAPTER II

INTRODUCTION TO THE REVIEW

The review of the literature has been divided into several sections. The first section deals with the component curve technique, its derivation and application. The second section is a justification for classifying the concept formation of individuals. The third and fourth sections review literature pertaining to cognitive and personality correlates of concept formation. The final two sections deal with complexity and transfer as they apply to this study. The description of learning through the use of graphical methods has attracted researchers in psychology chiefly because it is a concise yet easily interpretable means of illustrating a complex phenomena. It has been traditional to use the average learning curve of a group to determine the characteristics of the learning process.

Hilgard (1938), in discussing the Vincent procedure for averaging individual curves in order to reveal the form of the learning function, pointed out that by averaging the individual curves many of the variations due to individual differences are eliminated. He further pointed out that in many cases it is the individual differences that are of interest.

From a more mathematical point of view Hayes (1953) criticizes the use of averaging performances at successive

stages in the learning process on the grounds that the curves obtained are irrelevant inasmuch as their forms are determined not only by the form of their component individual curves but also by the distribution of the curves. That is to say, if there as a consistent majority group of individual curves the other curves would not be adequately described by the average curve. Perhaps an even more devastating criticism is that it would be possible to have an average learning that did not approximate any of the individual curves.

Estes (1956) in discussing this problem states that just as any mean score could have arisen from an infinite number of sampling populations so could a mean curve have arisen from an infinity of curve populations. Therefore no inductive inference from mean curve to individual curve is possible.

Estes divides learning functions into three classes in an effort to show what happens to the shape of learning functions when they are averaged.

Class A functions are of the form $y = ax + b$, $y = a \ln x$, $y = \frac{a}{x}$, and so on, wherein each parameter appears either alone or as a coefficient of the independent variable. In averaging class A curves, the form is unmodified and the parameters of the mean curve are the means of the parameters of the individual curves.

Class B functions are of the form $y = \log bx$ or $y = \frac{1}{a} + \frac{b}{ax}$, in these cases the form of the curve is unchanged

but interpretation of the parameters is complicated.

Class C functions are of the form $y = a + be^{-cx}$. In this case the form itself is modified by averaging.

Sidman (1952) suggests several alternatives to the problem of inference from mean to individual curves. One of these is to develop techniques which produce lawful individual functions. Such a technique has been developed by Tucker.

Tucker (1958, 1960) has shown that learning functions can be treated in the form $X_{ji} = a_{j1} y_{1i} + a_{j2} y_{2i} \cdots\cdots$ (1) where X_{ij} is the score of the "i"th individual on the "j"th task of a group of sequential learning tasks, where (i = 1, 2...n) and (j = 1, 2...N) $a_{j1} \ldots a_{jn}$ are trial parameters and $y_{1i} \ldots y_{Ni}$ are individual parameters. For class A and B functions there will be a few terms on the right hand side of equation (1) but for class C functions there will be an infinite series of terms.

The method devised by Tucker is based on the idea that equation (1) is identical to the basic linear postulate used in factor analysis. In factor analysis the observed score X_{ji} is said to be the weighted sum of the factor scores for the individuals, the weights being the factor loadings for the variables. In this case a sequence of scores for an individual can be considered to be the weighted sum of reference learning curves, the weights being the individual scores and the factor loadings forming the reference curves.

This technique is analagous to Fourier analysis in which complex waves are resolved into a series of simple sine waves. The factor component curves like the component curves arising from a Fourier analysis can be interpreted in such a way that they may represent either individually or in groups, particular processes. The two techniques are similar in another respect inasmuch as neither procedure has a unique solution.

Providing that the component curves can be interpreted meaningfully one can, by examining individual loadings on the reference curves, gain an understanding of the extent to which an individual's learning approximates the process implied by the reference curve.

The relationship between the loadings and certain personality and cognitive variables can be determined using the technique of canonical correlation. If the individual loadings tend to separate the subjects into groups, a discriminant function can be calculated using the cognitive and personality variables. Both of these methods will help to determine the underlying dimensions of the processes represented by the curves.

The derivation of these factor curves and component curves is given below. As in factor analysis the procedure amounts to finding components that will approximate in a least squares sense a given matrix. In this case the score matrix is to be approximated by the product of the individual

loadings and the reference curve matrix. An element of this matrix can be expressed as:

$$X_{ji} = \sum_{m=1}^{r} a_{jm} y_{mi} \quad \left(\begin{array}{ll} r < n & \text{when } n \leq N \\ & \\ r < N & \text{when } n \leq N \end{array} \right)$$

and $\sum (X_{ji} - \hat{X}_{ji})^2$ is a minimum for a given number (r) of factors used. The score matrix can be therefore written as $X = AY$.

In solving for A and Y the score matrix is written as the product of three matrices (equation 2) where U and W are orthonormal matrices of order (NxN) and (nxn) respectively. G is an Nxn matrix with $g_1, g_2, \ldots . g_N$ (when $N < n$) as elements in the first N upper left diagonal positions and zero elsewhere.

$$X = UGW \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Tucker calls the columns of U the left principal vectors and the rows of W the right principal vectors of X. Equation (2) is referred to as the canonical resolution of X (Eckart and Young 1936).

U and G can be found by finding the latent roots and vectors of the matrix XX'. This will give G^2 as the matrix of latent roots from which G can easily be found. In approximating the matrix X only the r largest roots are selected and equation (2) can be written as $X_r = U_r G_r U_r$. In this case U_r will be a matrix of order (N x r), W_r will be a matrix of order (r x n) and G will be a diagonal matrix of order (r x r).

Weitzman (1963) states that in the case where the number of individuals excedes the number of tests U_r can be called the co-ordinates of N learning sessions in r space, and W_r is a table of performance points of n subjects in the same space. He goes on to state that U_rG_r is equivalent to r factor component curves running over N learning sessions.

Each subject's learning curve is represented in approximation by a row of X_r and may be regarded as a weighted sum of factor component curves. The weights are given for each individual as columns of W_r. Since the simple sum of the factor component curve also represents a composite learning curve it should be of the same order of magnitude as the weighted sum. Considering for a moment only the matrix of weights W_r, it can be seen that because W_r is part of an orthonormal matrix W the sum of squares of the n possible entries in each row must be one. In order to make the weighted sum of the component curves the same order of magnitude as the simple sum the weights of W_r must be increased such that the sum of squares of the rows of W_r will approach the value n. This can be accomplished by multiplying the matrix W_r by $\sqrt{n}$ to obtain the individual weights. To preserve the equality of equation (2), UG must be multiplied by $\frac{1}{\sqrt{n}}$.

The component curves are given by the relationship

$$Z_r = n^{-\frac{1}{2}} U_r G_r \qquad (3)$$

and the individual weights are given by

$$B_r = n^{\frac{1}{2}}W_r \qquad (4)$$

The matrix B can be calculated more easily by using the following relationships $Z = n^{-\frac{1}{2}}UG$

From which $n^{\frac{1}{2}}G^{-1}Z' = U$

and $G^{-1}U^{1}X = W$

to form $B = nG^{-2}ZX$ from which $B_r = nG_r^{-2}Z_rX$ (5)

In comparing this solution to factor analysis, the matrix U is equivalent to the factors arising from an analysis of intercorrelations between tests. The matrix W is equivalent to the factors arising from an analysis of intercorrelations between people.

Since the solution of B_r and Z_r is not unique, B_r may be transformed by T such that $A_r = B_rT$ and $Y_r = T^{-1}Z_r$, where T is a non-singular transformation matrix the properties of which are described in the experimental procedure.

The determination of the number of significant component curves is a problem that is still unsolved. This is the same as determining the rank of X_r.

Weitzman (1963) suggests applying the runs test (see Seigel 1956, p. 52) to the columns of the matrix Z. It is desired to have a value of $r < N$ such that $(\hat{X}_r - X)$ can be considered to be random error. Since the first component curve accounts for the most variance and thus approximates the over-all mean curve, and since all the factor component curves are mutually orthogonal, the remaining component

curves should fluctuate about the zero line. Any significant trends in this fluctuation will be indicated by the runs test, where the runs consist of pluses and minuses representing fluctuations of the factor component curves above and below the zero line. The value of r then is one less than the number of the first factor component curve which shows insignificant trend and which may for this reason be regarded as an effect of the random error in X.

The procedure can be summarized into a few steps.

1. Formation of XX' where X is the observed score matrix.

2. Determination of the latent roots and vectors of the matrix XX' using Householder's method.

3. Calculation of the runs test to determine the number of significant curves.

4. Calculation of the trial parameters B_r.

5. Calculation of the individual weights Z_r.

6. Rotation of axii.

Justification for Classifying Concept Formation

Implicit in this study is the assumption that individual differences in concept formation can be classified. The following literature supports the validity of this assumption.

In the work of Bruner, Goodnow and Austin (1956) values of the attributes shape, color, number, and border were presented to a subject who formed an hypothesis about

the concept after being informed as to whether the concept carrying card was an exemplar of the concept that the examiner had in mind. As each card was presented, it either confirmed or infirmed the subject's previous hypothesis.

It was found that subjects tended to use one of two strategies in solving the task. These strategies were the wholist strategy and the partist strategy.

Goldstein and Sheerer (1941) using sorting tasks found what they described as concrete and abstract behavior. Hanfman and Kasanin (1942) support this view.

In discussing general approaches to research and theory in this area Vinacke (1952) describes two orientations. The first is the composite-photograph theory in which the problem solver passively acquires a picture of the common points. This view is held by Moore (1911), Fisher (1916) and others. In contrast to this is the active search theory advocated by English (1922) and others in which the subject makes an effort to solve the problem by making and testing hypotheses. Vinacke supports the idea that both theories are useful in explaining different kinds of behavior.

Yudin and Kates (1963), investigating concept formation in children, found three kinds of strategies. Working with university graduates, Blatt and Stein (1959) were able to classify problem solvers as being efficient and inefficient.

It can be seen therefore that there is some precedence in classifying problem solving behavior.

Cognitive Variables Affecting Concept Formation

The most extensive study in this area was done by Baggaley (1955) in which he had subjects sort cards into two piles. The cards had five dimensions, two attributes per dimension. He found the time required to make a decision with respect to each card's placement. After the subject had made his decision the experimenter indicated if it were correct or incorrect. It was found that there were two groups, analytic types and non-analytic types. Using log time as the dependent variable, the analytic types showed higher variability than the non-analytic types. It was further found for the whole sample that performance on the sorting task was correlated with strength of closure .45, deductive reasoning .31, figure analogies .36, inductive reasoning .26, and speed of closure .28. He found no correlation between log time and spatial relations, vocabulary, number word fluency or ideational fluency.

Harootunian and Tate (1960) investigated problem solving as a function of problem recognition, word fluency, ideational fluency, closure, intelligence, and reading. They found that reading and intelligence were the best predictors of problem solving. When these two variables were removed, problem recognition became the best predictor. The other variables were found to make little independent contribution.

Guilford (1954) states that there are several kinds of problem solving and for each kind of problem there are

usually several possible operations by which it can be solved. It is his opinion that general reasoning, fluency of ideas, flexibility, evaluation, and originality are among the abilities most likely to be important in problem solving.

In a factor analytic study of concept learning Stewart (1960) analyzed 15 reasoning tests, 13 reference tests, and 4 concept tasks (I. verbal sorting, II. figural sorting, III. figural memorizing, IV. verbal memorizing). Following oblique rotation, she found that the four tasks had loadings of greater than .25 on what she described as a flexibility of reasoning factor.

Staats (1957) found that scores on the abstract reasoning portion of the D.A.T. correlated -.33 with log time required to solve the two string problem.

In Maltzmann, Eisman and Brook (1956) study involving the two spheres problem, subjects above the median on the abstract reasoning test produced more solutions than those below.

McNemar (1955) used four types of reasoning items to select high and low reasoners from a large group of students. These students were compared on fluency, inductive reasoning, deductive reasoning, and variability as tested by the water jar problem. She found that the high reasoners were significantly better on accuracy and speed of induction and accuracy of deduction.

Osler and her associates (Osler and Fivel, 1961; Osler and Weiss, 1962), working with relationship of intelligence to problem solving found that when explicit instructions were given to normal and superior children there was no difference in their problem solving ability. When the children were required to find the problem, that is, there were no explicit instructions, the superior children were more successful.

In further work, it was found (Osler and Trautman, 1961) that increased dimensionality of the problem slowed the superior group but did not effect the normal child. This was explained on the basis of the normal children achieving solutions by a slow build up of S-R associations, while the superior students actively test hypotheses. This interpretation would be in line with that outlined by Vinacke (1957).

In studying memory and its effects on problem solving, Cahill and Hovland (1956), found that concept attainment is markedly superior at two levels of difficulty when all previous instances are exposed, compared to the case when only the current instance is exposed.

In summary these studies indicate that reasoning factors and memory are important in concept acquisition. Rather little work has been done on the relationship of these cognitive variables to the Brunerian type of concept tasks.

Personality Variables and Problem Solving

Little work has been done on the relationship of personality to problem solving. That which has been done indicates that this may be a fruitful area to investigate.

Blatt and Stein (1959) found correlations of the order of .35 between problem solving efficiency and the Esthetic score on the Allport Vernon Lindsay Scale.

Staats (1957) found a significant but small correlation (.11) between Taylor anxiety scale and log time in the two string problem.

Romanow (1958) using a verbal concept task found that high anxious people did less well than low anxious people, when interfering or competing responses were strong, that is, when the number of possible alternatives was large the high anxious people had more difficulty.

In other studies less related to the present study Fillenbaun and Jackman (1961) demonstrated the inhibiting effect of dogmatic personality on certain types of problems involving the temporary adoption of a belief system inconsistent with reality. Similarly, Neal (1959) found authoritarian personalities had more difficulty learning ambiguous material than non-authoritarian personalities.

Wesley (1953) found that anxious, rigid and normal subjects chosen on the Wesley Rigidity Inventory and the Taylor Manifest Anxiety Scale, took equally long to learn a concept task, although when the task was changed without

warning, the rigid group took longer to shift.

Rhine (1957) found no relationship between anagram solving and the McClelland achievement test.

Finally, Eysinck (1959) selected introverts and extraverts on the basis of the Maudsley Personality Inventory and found no significant difference in problem solving when the criteria were either total items correctly solved, or speed of solution. He did find, however, that extraverts took longer to obtain correct solutions at the end of the test and gave up more easily during this time.

As can be seen, investigation of the personality correlates of problem solving is in its infancy. There is controversy as to what effect, if any, the variables have on the process of concept formation. In reading the research, one is intuitively left with the feeling that the personality variables are important but that the means of investigating this importance have been inappropriate.

Task Difficulty and Concept Formation

A minor part of the present study is concerned with the relationship of task difficulty and component curve complexity. It is therefore appropriate to cite research on how varying degrees of difficulty can be achieved in Brunerian type tasks.

Bricker (1955) found that stimulus redundancy, retarded learning by making the stimulus patterns more complex and hard to identify. That is, if only two examples

are necessary in identifying the concept, a third but redundant example will tend to increase the difficulty of the task.

Reed (1946), using verbal problems, agreed with the above finding and further found that, as the complexity of the stimuli increased, there is a definite trend to shift from logical to illogical learning.

As cited earlier, Osler and Trautmann (1961) found that the difficulty of a concept task was increased by increasing the number of relative dimensions.

Wallach (1962), Shepherd, Hovland, and Jenkins (1961) all agree that difficulty can be increased by increasing the number of relevant attributes.

In this study, therefore, difficulty of the task will be varied by varying the number of relevant attributes.

Transfer and Concept Formation

Eckstrand and Wickens (1954) found that subjects trained to learn concepts involving one attribute, for example color, solved problems involving that dimension more quickly than groups solving problems involving a dissonant dimension, for example form. This may be an indication that set plays a more important role than a "learning how to learn" function. However, Olsen (1963) has shown that subjects solving two problems did as well as subjects solving several problems. This lends support to the idea that there is some effect in "learning how to learn" which in this case took place in the first problem solved.

Shepherd, Hovland and Jenkins (1961) found that when problems were done in type blocks and a high level of mastery was required, transfer occurred.

A purpose of the present study is to show the effect on the component curves of transfer or "learning how to learn." In accordance with the above research, two tasks are used which involve the same level of difficulty, one level of one attribute, and which do not involve the same attributes. This latter condition is an effort to separate set from transfer.

CHAPTER III

DEFINITION AND HYPOTHESES

Concept Formation

For the purpose of this study, concept formation is assumed to be a process of categorization.

Being shown exemplars and non-exemplars and classifying other stimuli on this basis is considered a concept formation task. It is assumed that a concept has been formed when the subject can correctly classify twenty-four consecutive stimuli, or when the subject names the relevant attributes of the concept.

Difficulty

The task difficulty in accordance with the findings of Shepherd, Hovland and Jenkins (1961) is a function of the number of relevant attributes in a concept problem.

Complexity of the Learning Function

Tucker (1960) states that the basic learning function may be thought of as having a complexity equal to the number of reference curves included in it.

Transfer

In this study, transfer is restricted to using techniques developed in one concept learning task to facilitate learning in another.

HYPOTHESES

I. Individuals can be separated into learning groups according to their loadings on rotated factor curves.

II. These groups will differ on cognitive and personality variables.

III. A "learning how to learn" function can be demonstrated by increased slope in the component curves, and by earlier acceleration in the component curves on successive tasks.

IV. There is a relationship between task difficulty and complexity of the learning function, such that, as the difficulty of the task increases the complexity of the learning function increases.

CHAPTER IV

I. THE SAMPLE

Two classes of grade seven students at Mount Carmel School in Edmonton were used. The first class of thirteen boys and ten girls was used to test hypotheses I, II, and III, and the second class of eighteen girls and nine boys was used to test hypothesis IV. The concept problem was administered in May and the testing carried out in June, 1964.

II. THE TASK

A population of concept carrying cards was evolved using two attributes--shape and shading. Each attribute had three values. The three shapes were: square, triangle, and circle. The three shadings were white, black, and striped. On each card three figures were drawn and shaded so that all possible combinations of the values of the two attributes were considered. The description and number of each card is given in Appendix B.

Tasks were formed by selecting eighty-one cards from the population of cards. The selection was carried out as follows. It was first decided what the relevant attribute would be. Since it was desirable to have two tasks of

equivalent difficulty, two tasks of one value of each attribute were selected. For task one a card considered to be an exemplar was any card carrying a shaded square. For task three the concept was a black triangle. As it was necessary to vary the difficulty, using the criterion of Shepherd et al (1961) two values of one attribute, shown conjunctively were used as concept two. This involved all cards with a circle and a square.

In forming the task it was decided to break the task into ten sub-tasks of eight cards. Within each sub-task it was decided to have two exemplars and six non-exemplars. It was recognized that there is some controversy on the usefulness of non-exemplars and the role that they play in confounding the difficulty of the task (see Smoke (1933), Bricker (1955) and Shore and Sechrest (1961), therefore the ratio was kept constant for all tasks.

In setting each task up, the population of cards was divided into two groups, those cards which carried the relevant stimuli and those which did not. Twenty-one cards were selected for the exemplar groups using a table of random numbers, and sixty cards were selected for the non-exemplar group.

For each sub-task of eight cards, a table of random numbers was used to randomize the position of the exemplars within the sub-task. Twenty of the exemplars were then assigned positions in the task on the basis of their order of selection for the group of exemplars and the non-exemplars

were assigned to positions in the same order. The extra exemplar was used as an example card. Appendix C gives the make-up of the tasks.

An example task consisting of eighteen cards was prepared using the concept triangle. In this case seven exemplars and eleven non-exemplars were used.

III. TESTS USED

Holzinger-Crowder Uni-Factor Tests (1958) Form Am

It is claimed by the authors (1958) that this test measures four types of mental activity: Verbal, Spatial, Numerical, and Reasoning. An intelligence quotient can be derived from these measures by referring a weighted composite score to a table provided by the publisher.

The factorial validity is inferred from the correlations between factor scores and school achievement in various subject areas. The correlation between the verbal factor and mathematics, science, social studies, and English, ranges from .51 to .73. The correlation between the spatial factor and the above subjects is small, the highest relationship in this area being with industrial arts. The correlation between the numerical factor and mathematics is .53.

Junior-Senior High School Personality Questionnaire (HSPQ)

According to the authors, Cattell, Beloff, and Coan (1958) the HSPQ is a test designed to cover all major

dimensions involved in individual differences in personality. This test was selected because it deals with a wide range of personality variables. Since rather little research has been done in the area of personality correlates of concept formation it was hoped that such a test would point the way for more extensive study. An outline of the fourteen scales is given in Table I, and is adapted from the test manual.

Memory Task

This task was designed to be used to investigate the role of memory in this type of concept formation task. An attempt was made to measure as nearly as possible the type of memory that could be involved in the concept formation tasks.

The test consisted of six cards selected from the population of concept cards. The cards selected were numbers 8, 38, 138, 123, 10, 27. These cards were exposed for fifteen seconds each. Following the last exposure the students were required to draw the cards in the order of presentation. A point was given for each figure correctly drawn on each card. The total score therefore was eighteen. A further test of seven cards was given. These cards were numbers 6, 87, 109, 151, 95, 61, 112. Exposure was the same as the initial task. The students were required to draw the cards but the order of answering was not important. One point was given for each correctly drawn card.

TABLE I

TITLES AND DESIGNATIONS OF THE FOURTEEN PERSONALITY DIMENSIONS

Variable Number	Cattell's Designation	Popular Title
1	A	Stiff, Aloof - Versus - Warm, Sociable
2	B	Dull - Versus - Bright
3	C	Emotional, Immature, Unstable Versus - Mature, Calm
4	D	Stodgy - Versus - Unrestrained
5	E	Mild - Versus - Aggressive
6	F	Sober, Serious - Versus - Enthusiastic, Happy-Go-Lucky
7	G	Casual, Undependable - Versus - Conscientious, Persistent
8	H	Shy, Sensitive - Versus - Adventurous, "Thick-Skinned"
9	I	Tough, Realistic - Versus - Esthetically Sensitive
10	J	Liking Group Action - Versus - Fastidiously Individualistic
11	O	Confident - Versus - Insecure
12	Q_2	Group Dependent - Versus - Individually Resourceful
13	O_3	Uncontrolled, Lax - Versus - Controlled, Showing Will Power
14	Q_4	Relaxed, Composure - Versus - Tense, Excitable

Note: Adapted from Cattell, Beloff and Coan (1958, p. 6.)

The memory score was derived by addition of the two scales making a possible score of twenty-five.

It was hoped that a test could be derived that would closely approximate the memory requirements of the concept task. This was not possible. On one hand it was decided that the concept may involve remembering separate figures on the card. The only way to test this was to make the order of presentation important. This eliminated guessing to some extent. On the other hand in the concept task it would not be necessary to remember the order of presentation of the cards. For this reason the second section of the memory test was added.

IV. EXPERIMENTAL PROCEDURE

Task Presentation

The two groups used were separated on the basis of classroom divisions for ease of task presentation.

Group A was used primarily to test hypotheses one, two, and three. Group B was used to test hypothesis four.

Group A was administered tasks one and three according to the instruction in Appendix A. Group B was administered tasks three and two according to the same instructions.

The students were required to respond to each card in three ways:

1. Indicating whether the card carried the concept.

2. Indicating by checking one of the shapes and one of the shadings which attributes they thought were important.

3. Indicating how sure they were that the concept they had selected was correct.

The answer sheet is included in Appendix D.

Following the administration of each task the students were asked to write down what they thought the correct concept was.

Analysis

Throughout the analysis the IBM 1620 Computer of the University of Alberta was used to facilitate the computations.

Each task consisted of eighty responses which were divided into ten sub-tasks of eight consecutive responses. The observed score matrix for Group A was of order (10 X 23) and for Group B was of order (10 X 27). For each task of each group the score matrix was multiplied by its transpose. The eigenvalues and eigenvectors of the sums of squares and cross products matrix was found using the Householder method.

The number of significant factors was calculated using the "Runs Test" as advocated by Weitzman (1963).

The trial parameters was calculated according to formula (3). The individual weights were calculated according to formula (5).

Rotation

The individual weights were plotted simultaneously for the significant factors and the axii were rotated orthogonally to maximize the discrimination between groups that appeared. This analysis was carried out entirely on a subjective basis.

The trial parameters were rotated to the same angular distance as the individual weights.

Test Presentation and Analysis

The objective tests were presented at one sitting to Group A in the following order: Memory tests, Holzinger-Crowder, Personality Inventory.

The matrix of inter-correlations between all the test variables and the factor loadings on the component curves was calculated. From this matrix certain tests were selected for more intensive study. The bases of selection were twofold, either the predictor (test) variables correlated with one or more of the factors or it appeared that it could act as a suppressor variable.

The subjects of Group A, Problem I, were divided into groups on the basis of their factor scores and a discriminant function analysis was carried out to separate the groups on the basis of the selected predictors.

A canonical correlation was carried out using the method outlined in Cooley and Lohnes (1962). This was done in order to determine the relationship between the

predictor variables and the factor scores.

For Group A, inter-correlations between factor scores for problems one and two were calculated to aid in the determination of the effect of transfer. To this end also a descriptive investigation of the "sureness" and "method" of concept attainment was carried out using the information on the answer sheets that indicated which attributes the subject deemed important, and how sure he was of the correct answer.

The relationship between task difficulty and complexity of the component curves was investigated descriptively using the "sureness" responses to the cards. The number of component curves necessary for each problem was calculated and the shape of the curves compared.

CHAPTER V

THE RESULTS

The matrices of sub-task scores are given in Tables XIV, XV, XVI, and XVII of Appendix E. The first five eigenvalues and their associated vectors are given in Tables XVIII, XIX, XX, and XXI of Appendix E.

Table II shows the calculations for the runs tests to determine the number of significant factors. The calculations were made on the basis of the signs of the elements in each eigenvector. Only those eigenvectors that showed a significant non-random trend were chosen. Tables found in Seigel (1956) were used to assess the significance.

From Table II, it can be seen two component curves are necessary for an approximation of the score matrices for Group A, while only one curve is necessary for each problem of Group B.

The unrotated loadings for the curves are tabulated in Tables XII, XIII, XIV, and XV of Appendix E and the associated individual loadings are given in Tables XVI, XVII, III, and IV.

The unrotated reference curves were constructed from the unrotated loadings and appear in Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. In each case one more than

the necessary number has been drawn for comparison.

For Group A, the curves were rotated in an effort to maximize the discrimination between the groups that appeared. Figure 11 shows a plot of the individual loadings on the two component curves for problem one. Similarly, Figure 12 shows the plot for problem two. The dotted lines indicate the directions of the rotated axii.

For problem one an angular rotation of 34.5 degrees appeared to give an optimum separation between individuals, whereas for problem two 41.5 degrees seemed appropriate.

The rotated reference curves are plotted in Figures 13,14,15,16. Since neither problem in Group B required more than one curve rotation was not possible. Individual loadings on the rotated curves are given in Tables V and VI.

As a means of finding out whether individuals could be separated on the basis of their loadings on the component curves, they were placed into groups and Hotelling's T^2 (Winer 1962) test was used to determine if the groups were significantly different with respect to individual loadings. All subjects who did not attain the concept were placed in group three. All other subjects were placed into groups one or two depending on which curve they loaded more highly on. The group membership is given in Tables V and VI.

TABLE II

CALCULATION OF RUNS TESTS FOR DETERMINATION OF THE NUMBER OF SIGNIFICANT CURVES

Group A

	Problem 1 Vector						Problem 2 Vector				
	1	2	3	4	5		1	2	3	4	5
Runs	1	2	3	5	8	Runs	1	2	3	6	9
N1[*]	10	5	6	7	5	N1	10	7	5	5	5
N2	0	5	4	3	5	N2	0	3	5	5	5
Significance	.05	.05	NS	N.S	N.S	Significance	.05	.05	N.S	N.S	NS

Group B

	Problem 1 Vector						Problem 2 Vector				
	1	2	3	4	5		1	2	3	4	5
Runs	1	5	4	7	4	Runs	1	3	6	8	8
N1[*]	10	4	7	3	6	N1	10	6	6	5	6
N2	0	6	3	7	4	N2	0	4	4	5	4
Significance	.05	N.S	N.S	N.S	N.S.	Significance	.05	N.S	N.S	N.S	N.S

[*] N1 is the number of positive elements in the eigenvector.

N2 is the number of negative elements in the eigenvector.

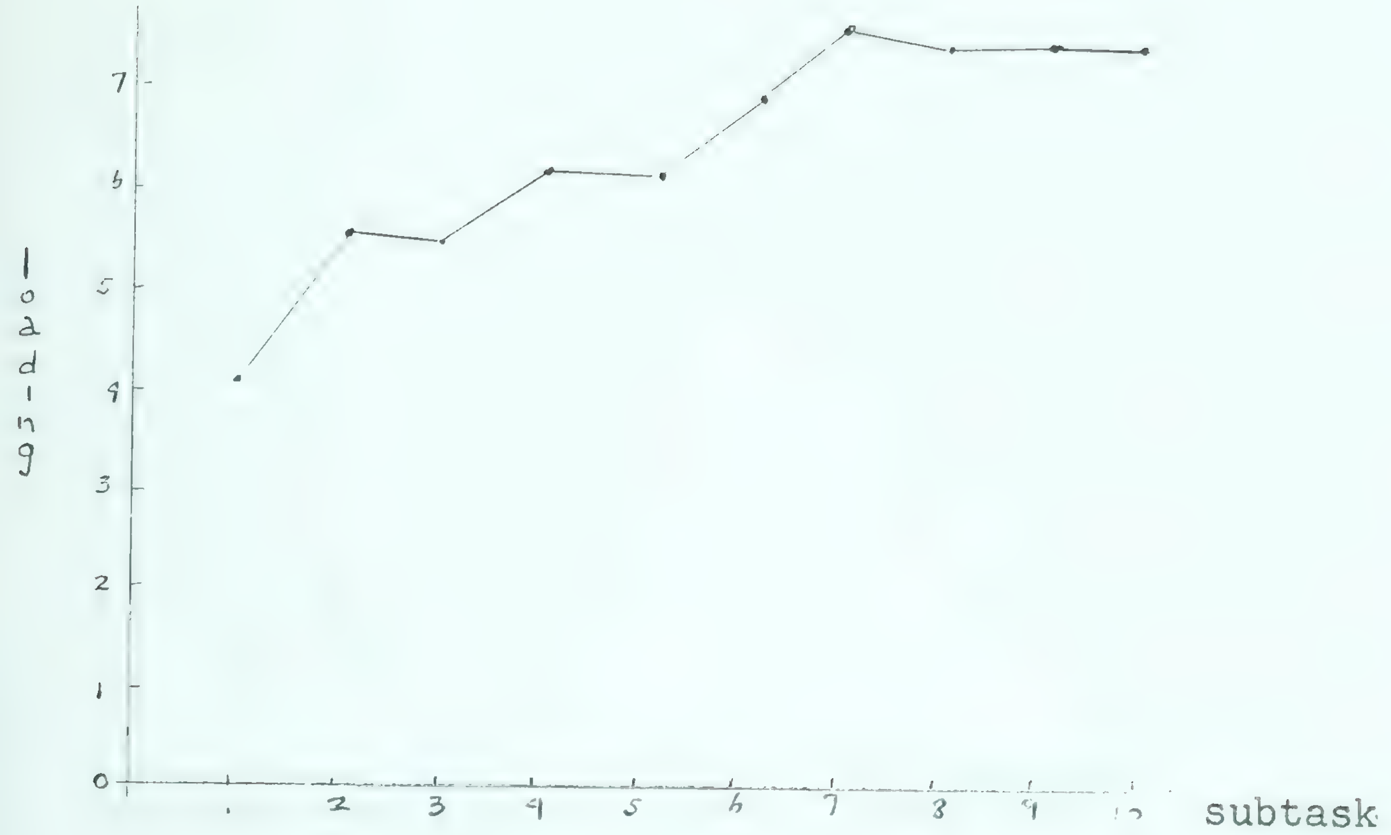


Figure 1

First Unrotated Component Curve, Group A, Problem One

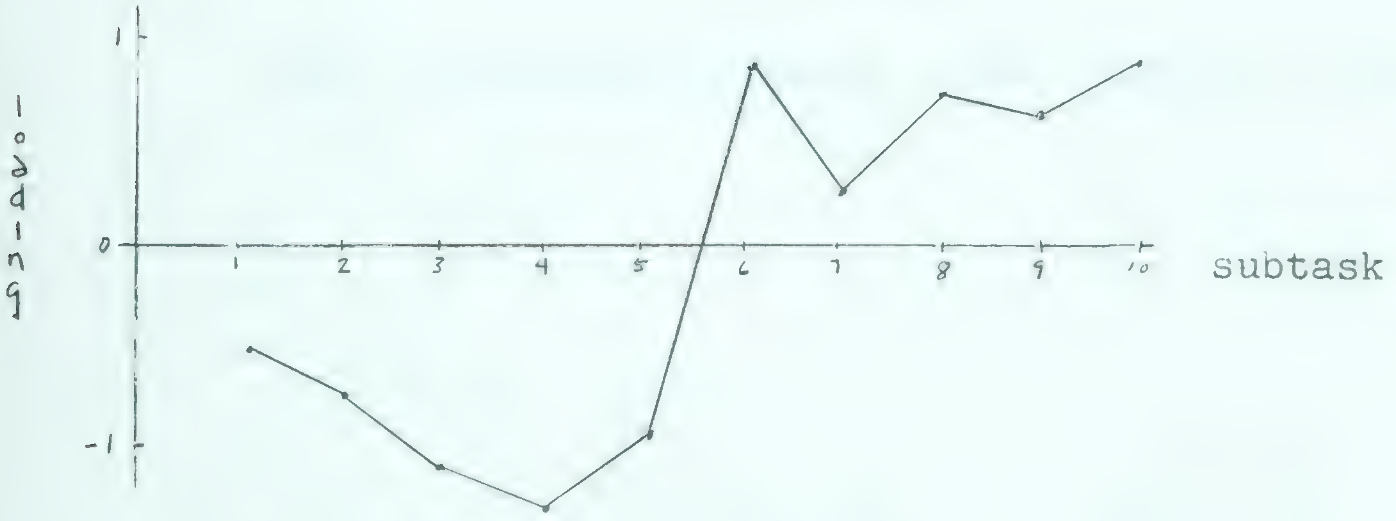


Figure 2

Second Unrotated Component Curve, Group A, Problem One

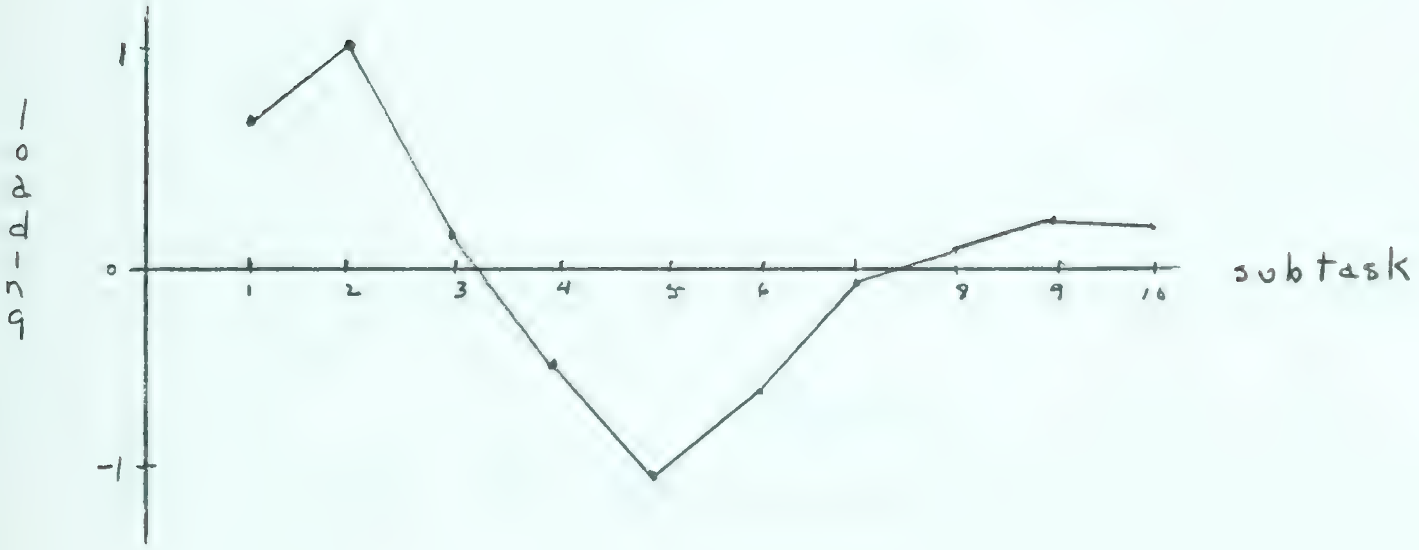


Figure 3

Third Unrotated Component curve, Group A, Problem One

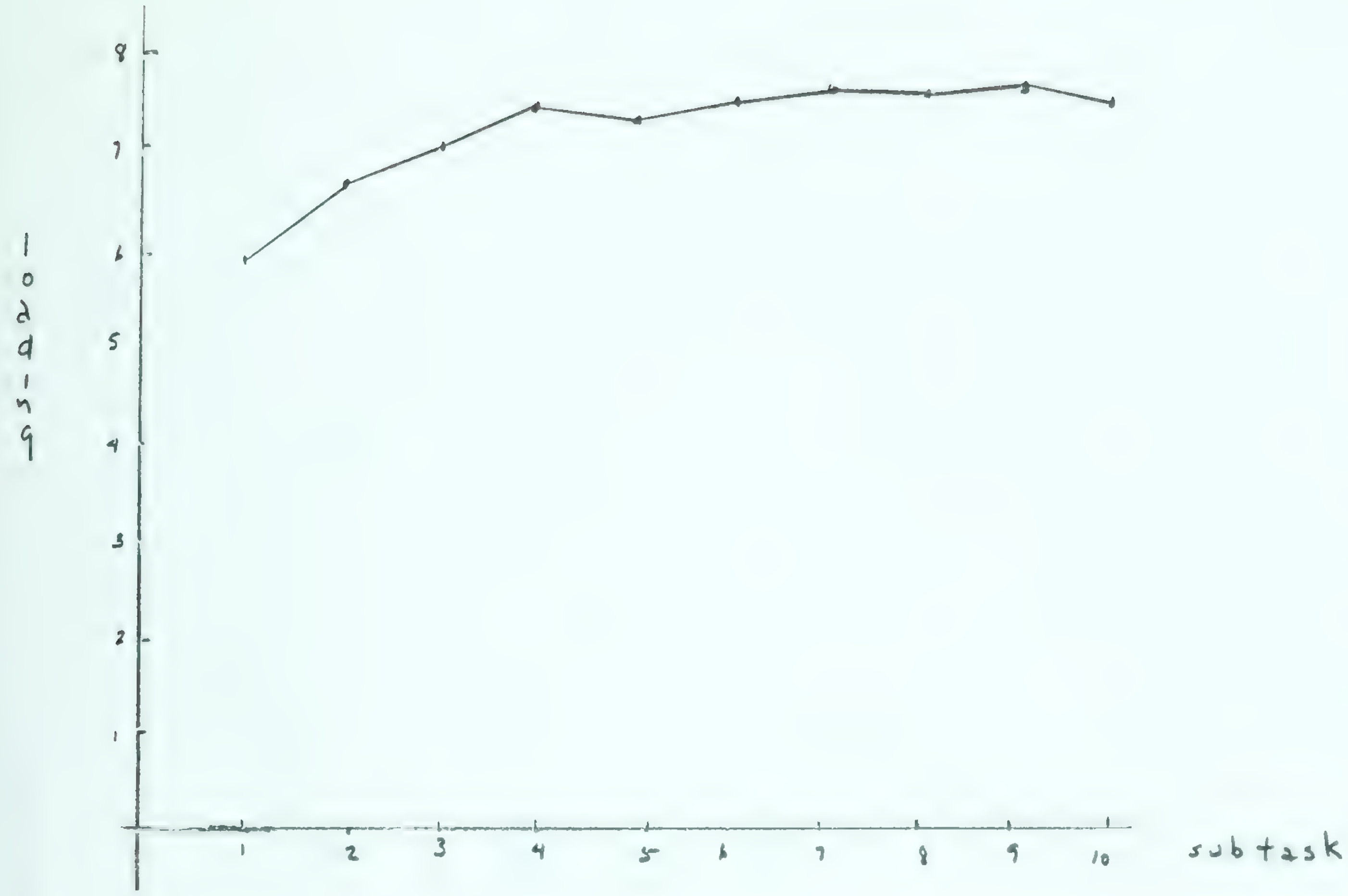


Figure 4

First Unrotated Component Curve, Group A, Problem Two

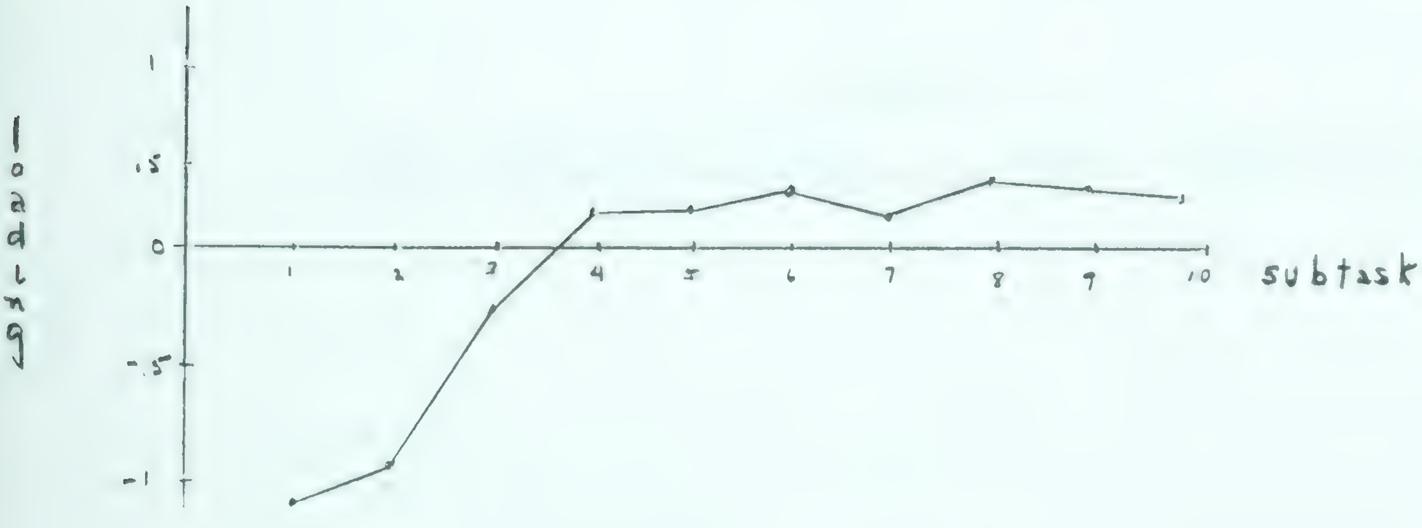


Figure 5

Second Unrotated Component Curve, Group A, Problem Two

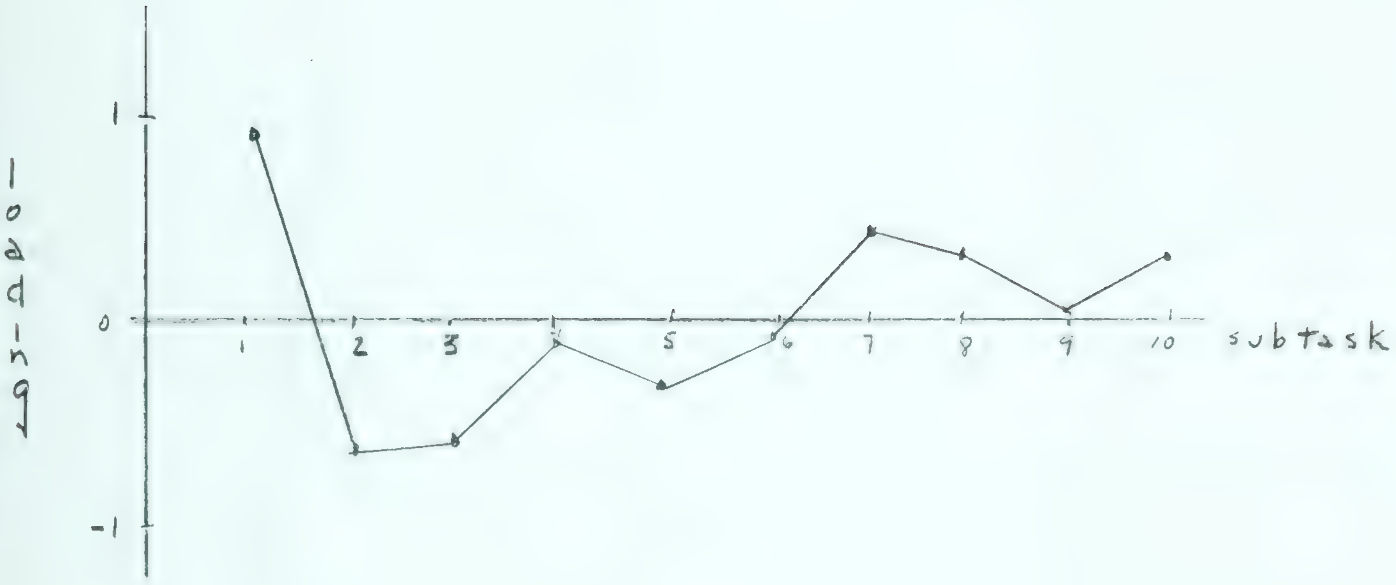


Figure 6

Third Unrotated Reference Curve, Group A, Problem Two

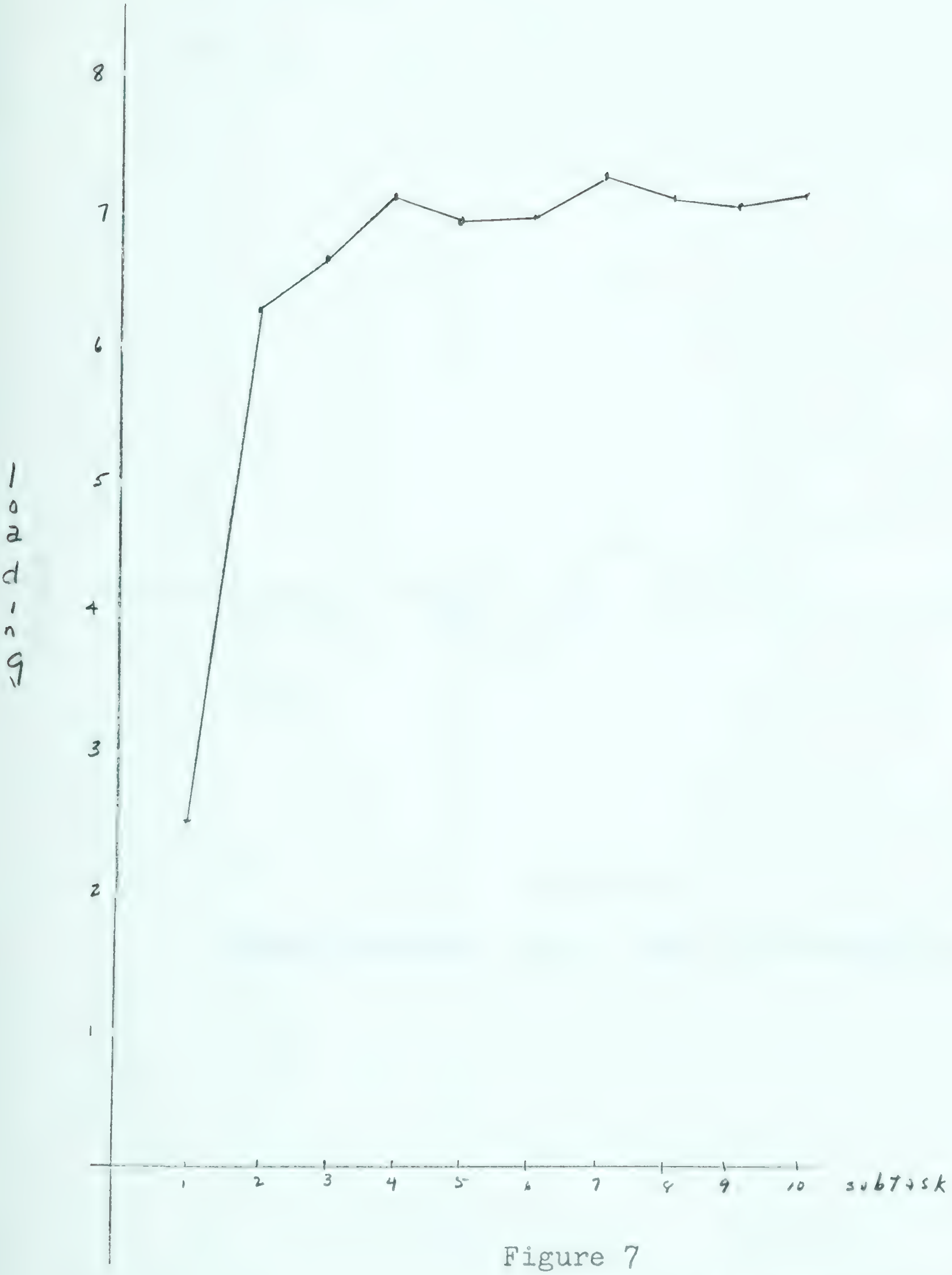


Figure 7

First Reference Curve, Group B, Problem One

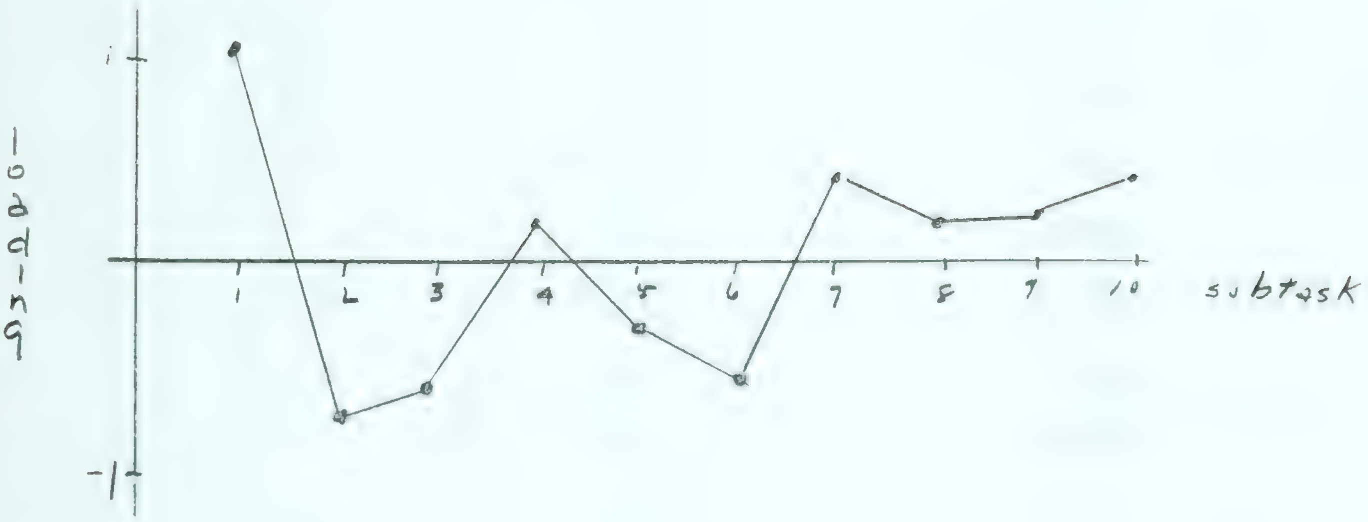


Figure 8

Second Reference Curve, Group B, Problem One

TABLE III

UNROTATED INDIVIDUAL LOADINGS, GROUP B

PROBLEM ONE

Individual	
1	.669
2	1.152
3	1.115
4	.676
5	1.056
6	.854
7	1.130
8	.883
9	1.144
10	1.200
11	.619
12	1.144
13	1.021
14	1.144
15	1.099
16	1.144
17	1.144
18	.571
19	1.135
20	1.124
21	1.112
22	.862
23	1.088
24	.577
25	1.152
26	1.130
27	.512

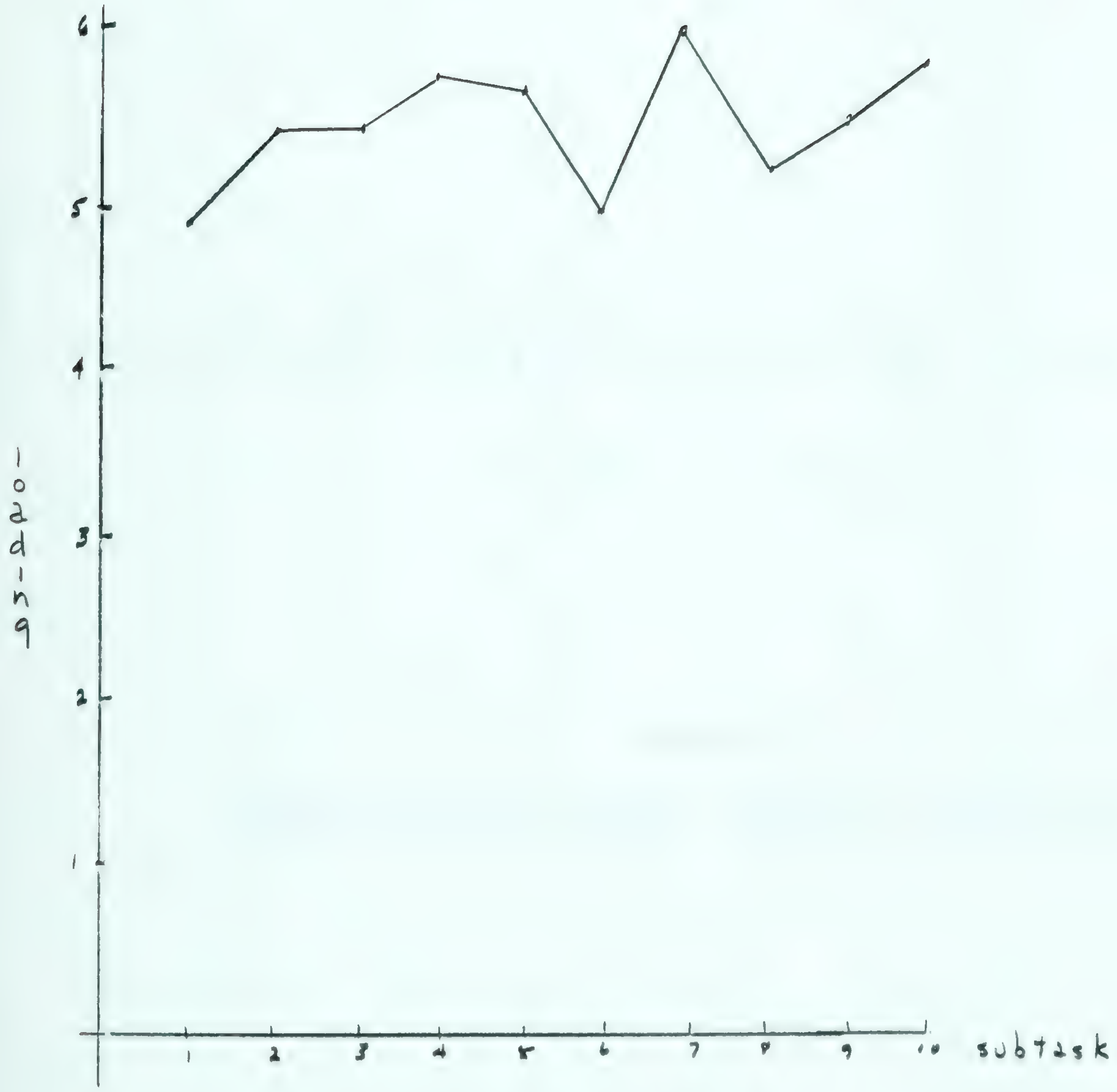


Figure 9

First Reference Curve, Group B, Problem Two

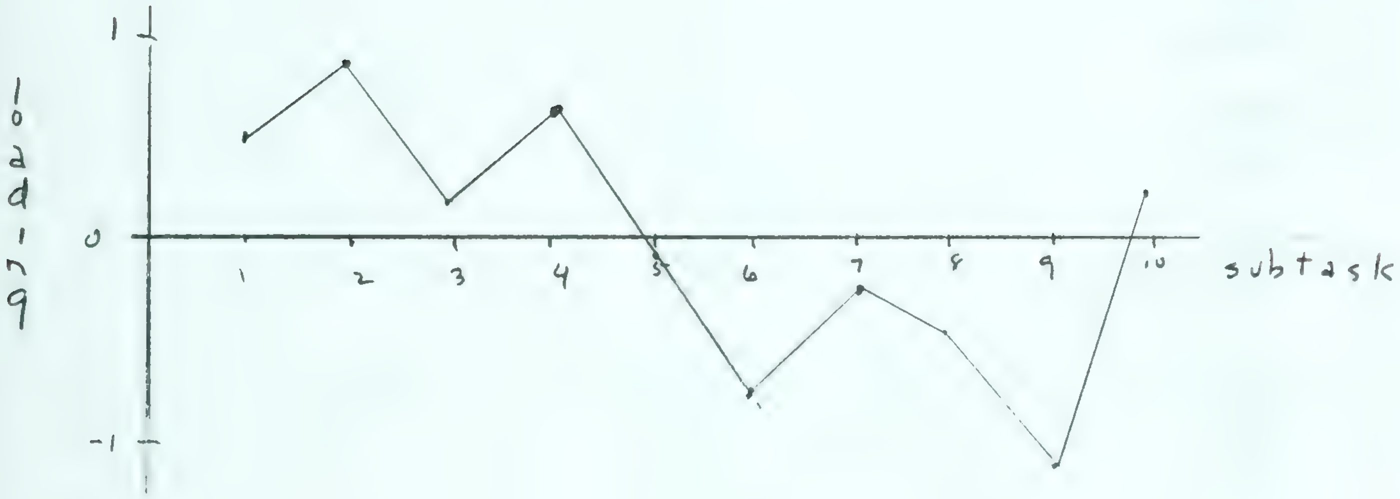


Figure 10

Second Reference Curve, Group B, Problem Two

TABLE IV

UNROTATED INDIVIDUAL LOADINGS, GROUP B

PROBLEM TWO

Individual	
1	1.107
2	1.299
3	.992
4	.853
5	.823
6	1.089
7	.929
8	1.108
9	.690
10	.999
11	1.177
12	.910
13	1.101
14	.927
15	.949
16	.963
17	.917
18	.848
19	.988
20	1.129
21	.866
22	1.200
23	.740
24	.990
25	1.281
26	1.007
27	.809

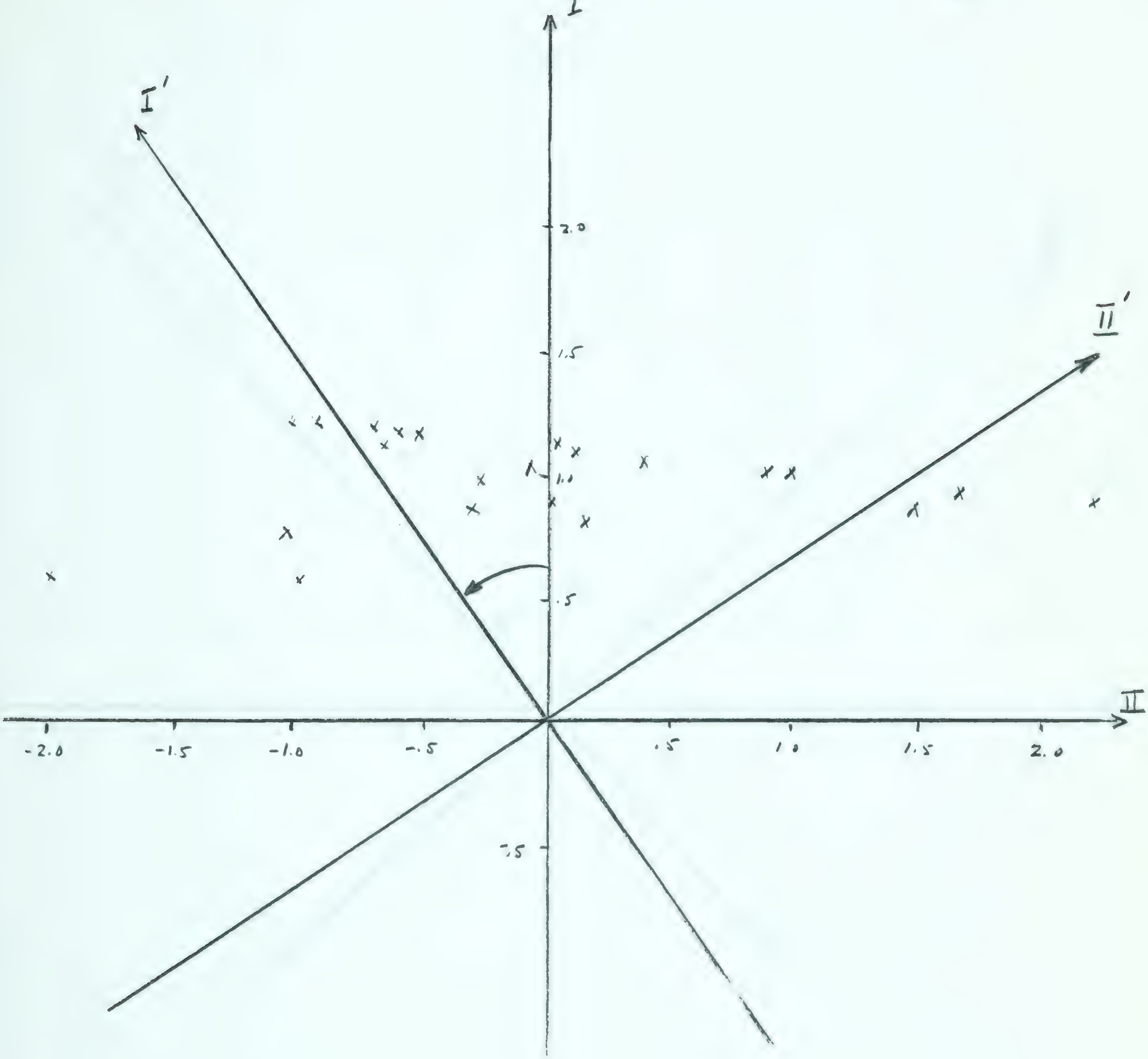


Figure 11

Interfactor Loadings, Group A, Problem One

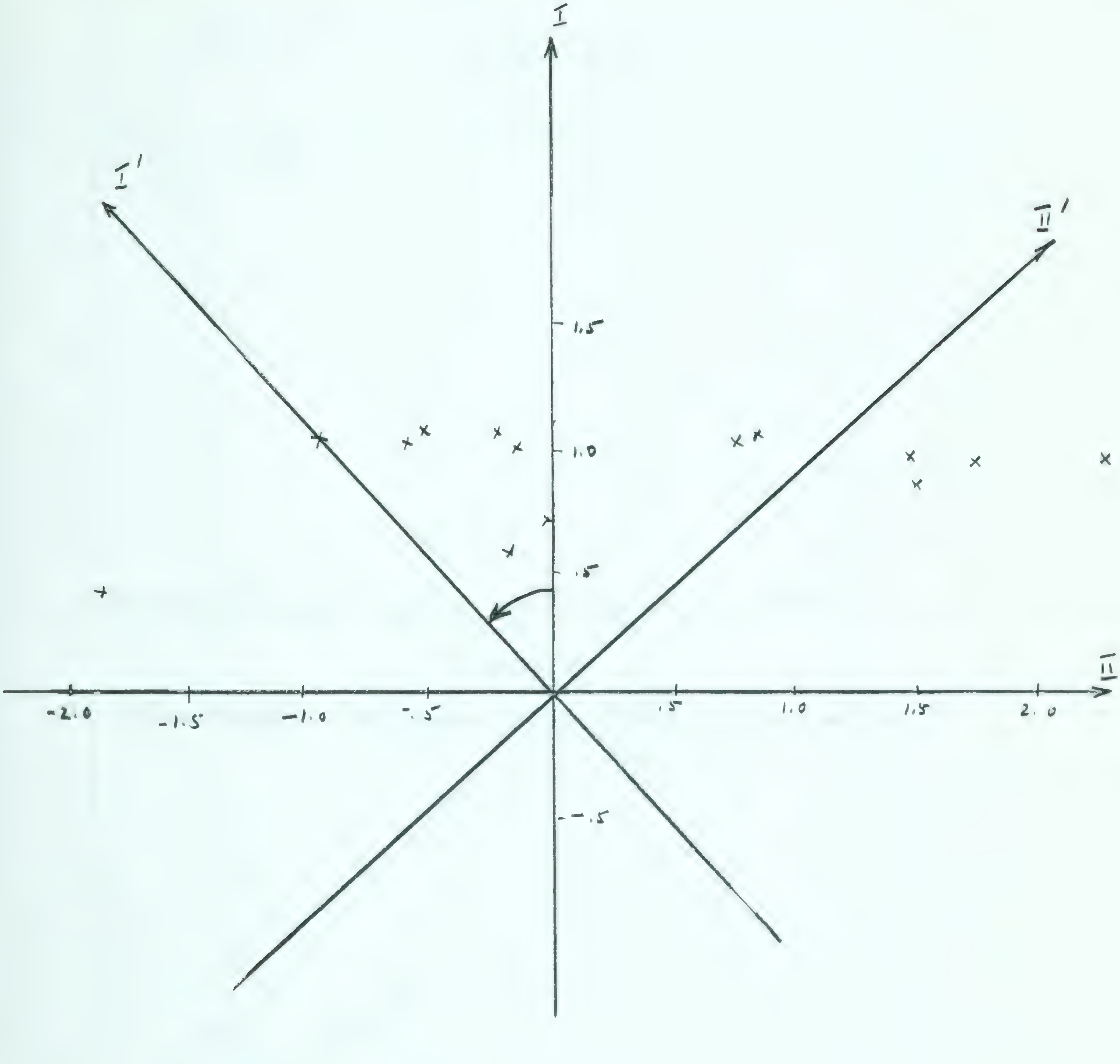


Figure 12

Interfactor Loadings, Group A, Problem Two

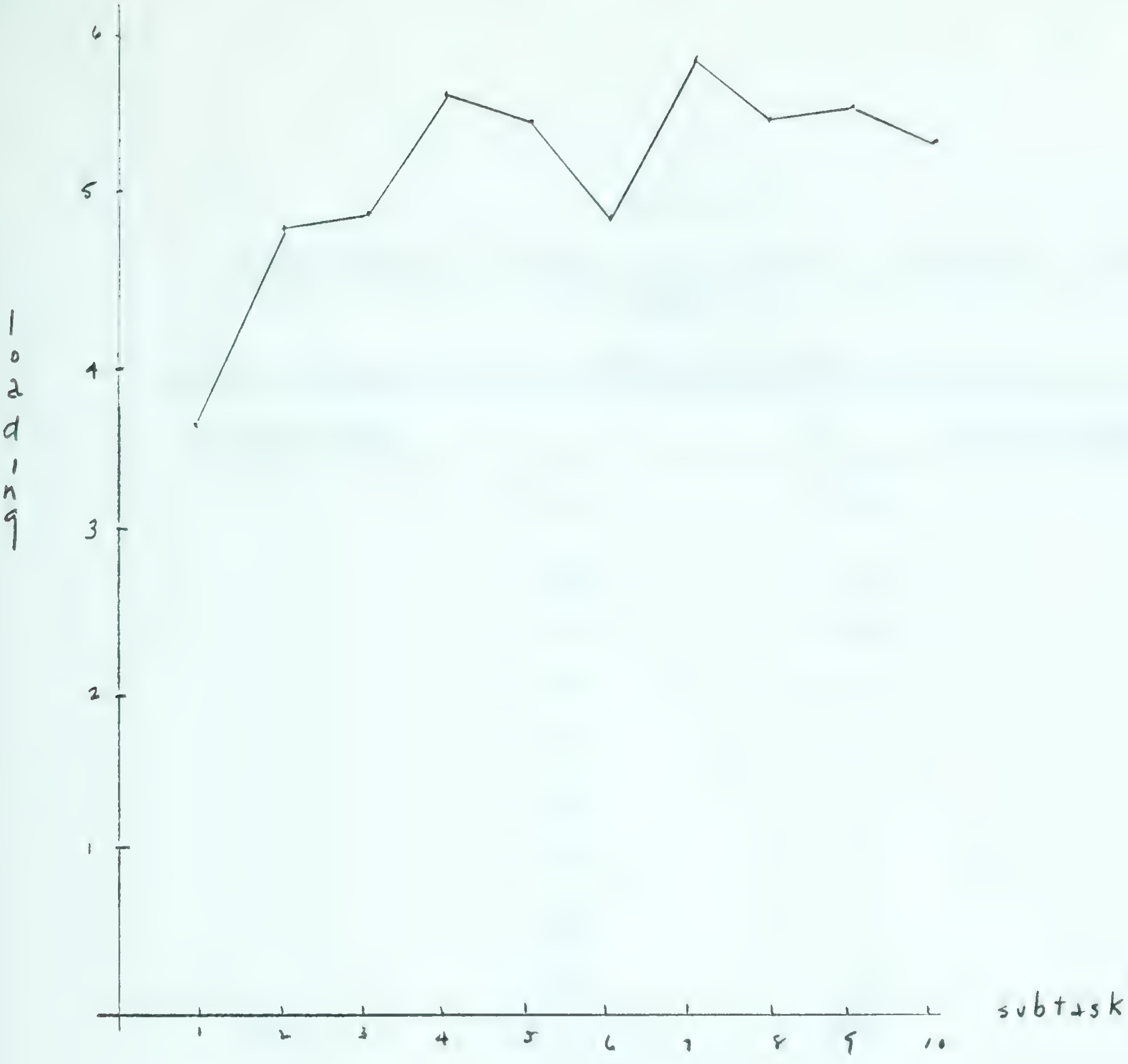


Figure 13

First Rotated Reference Curve, Group A, Problem One

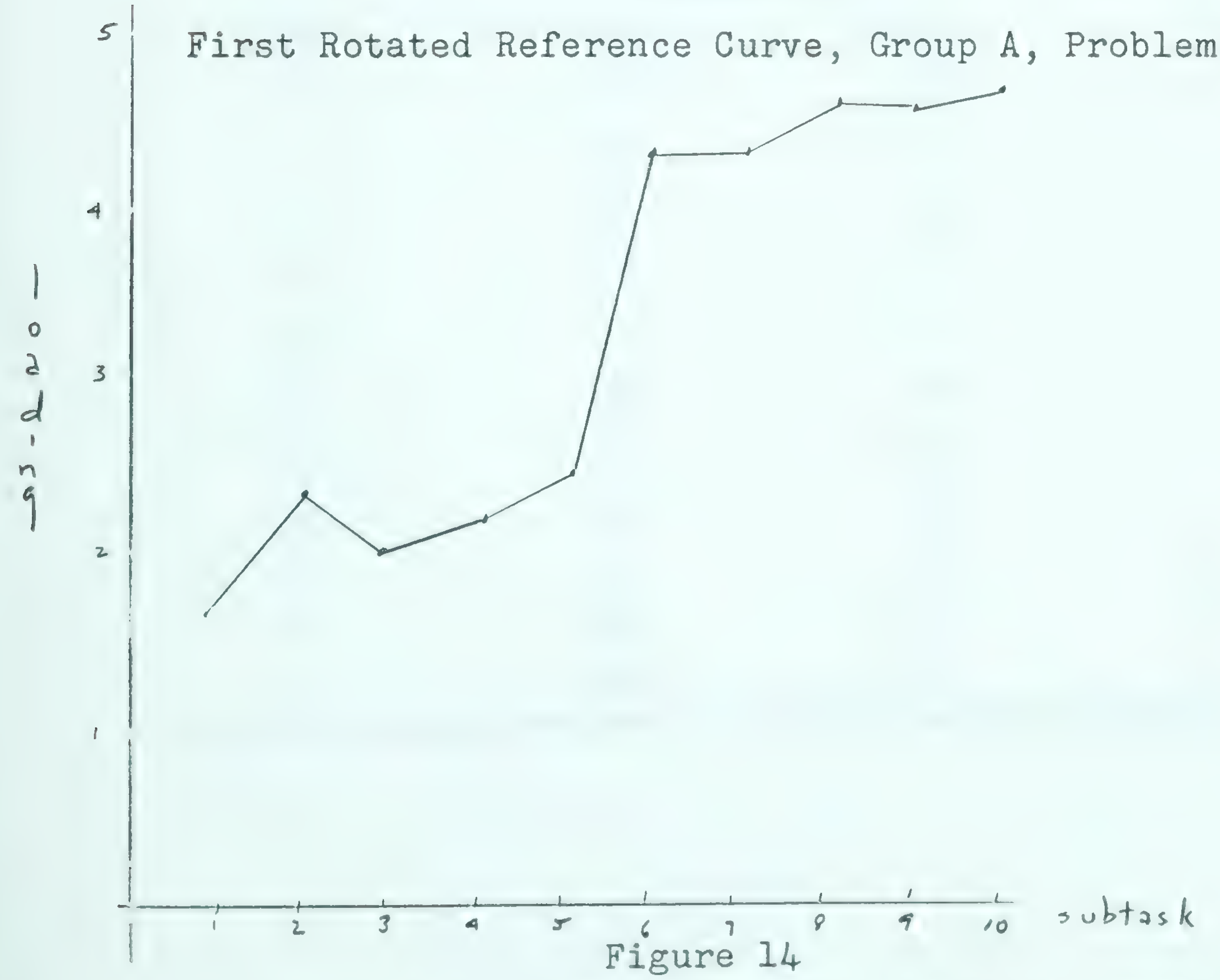


Figure 14

Second Rotated Reference Curve, Group A, Problem One

TABLE V

INDIVIDUAL LOADINGS ON ROTATED REFERENCE CURVES GROUP A

PROBLEM ONE

Individual	I	II	Group Membership
1	1.34	.08	1
2	1.60	-.18	1
3	.91	.66	1
4	1.20	-.45	3
5	1.29	.15	1
6	1.03	-.50	3
7	1.24	.23	1
8	.88	.51	1
9	1.60	-.18	1
10	1.53	-.18	1
11	1.53	-.10	1
12	-.13	1.71	2
13	.86	.23	3
14	-.50	2.36	2
15	.93	.31	1
16	.05	1.63	2
17	.33	1.33	2
18	.58	.58	2
19	1.60	-1.33	3
20	.64	.93	2
21	.73	.51	3
22	.28	1.39	2
23	.86	.70	1

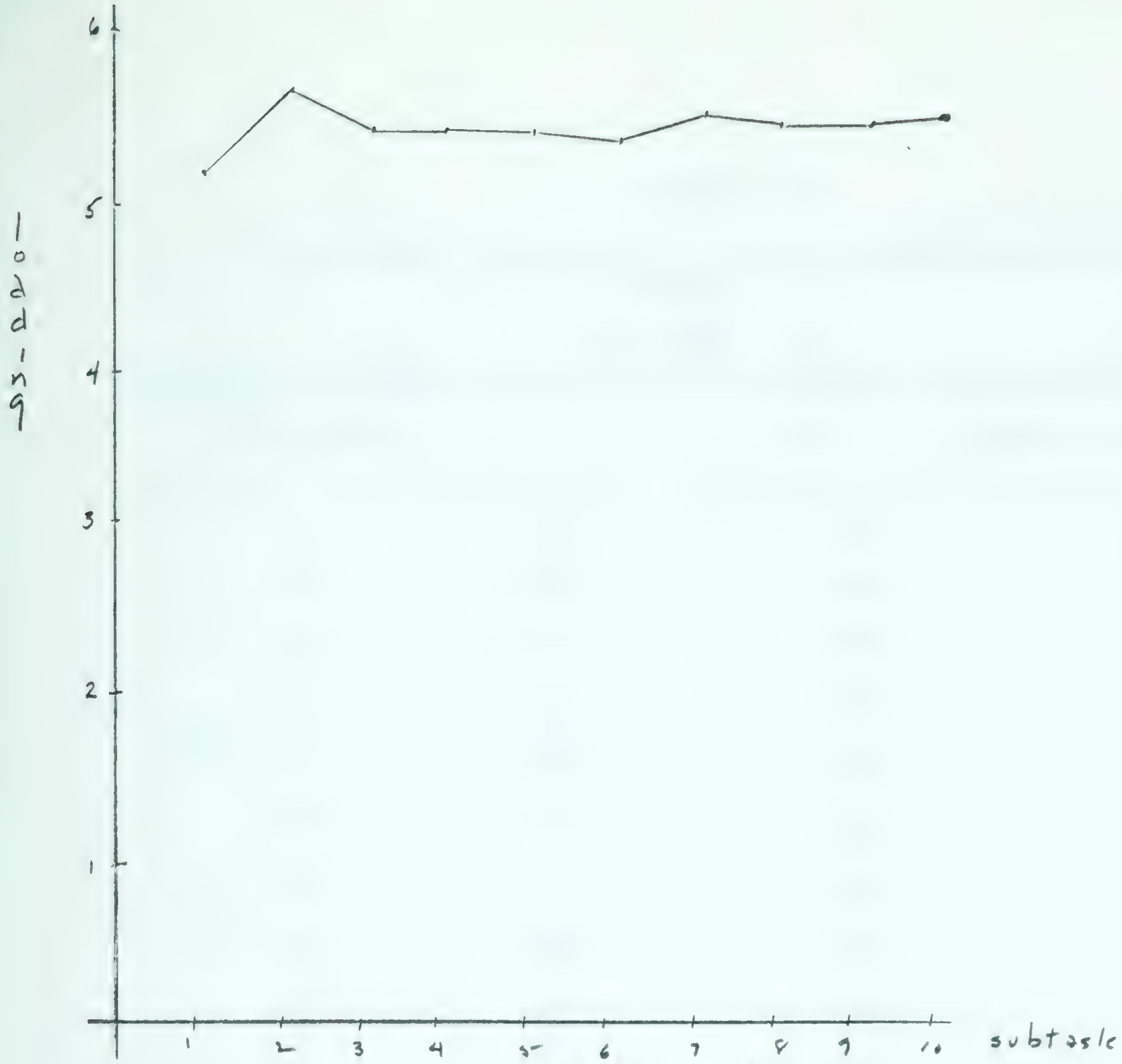


Figure 15

First Rotated Reference Curve, Group A, Problem Two

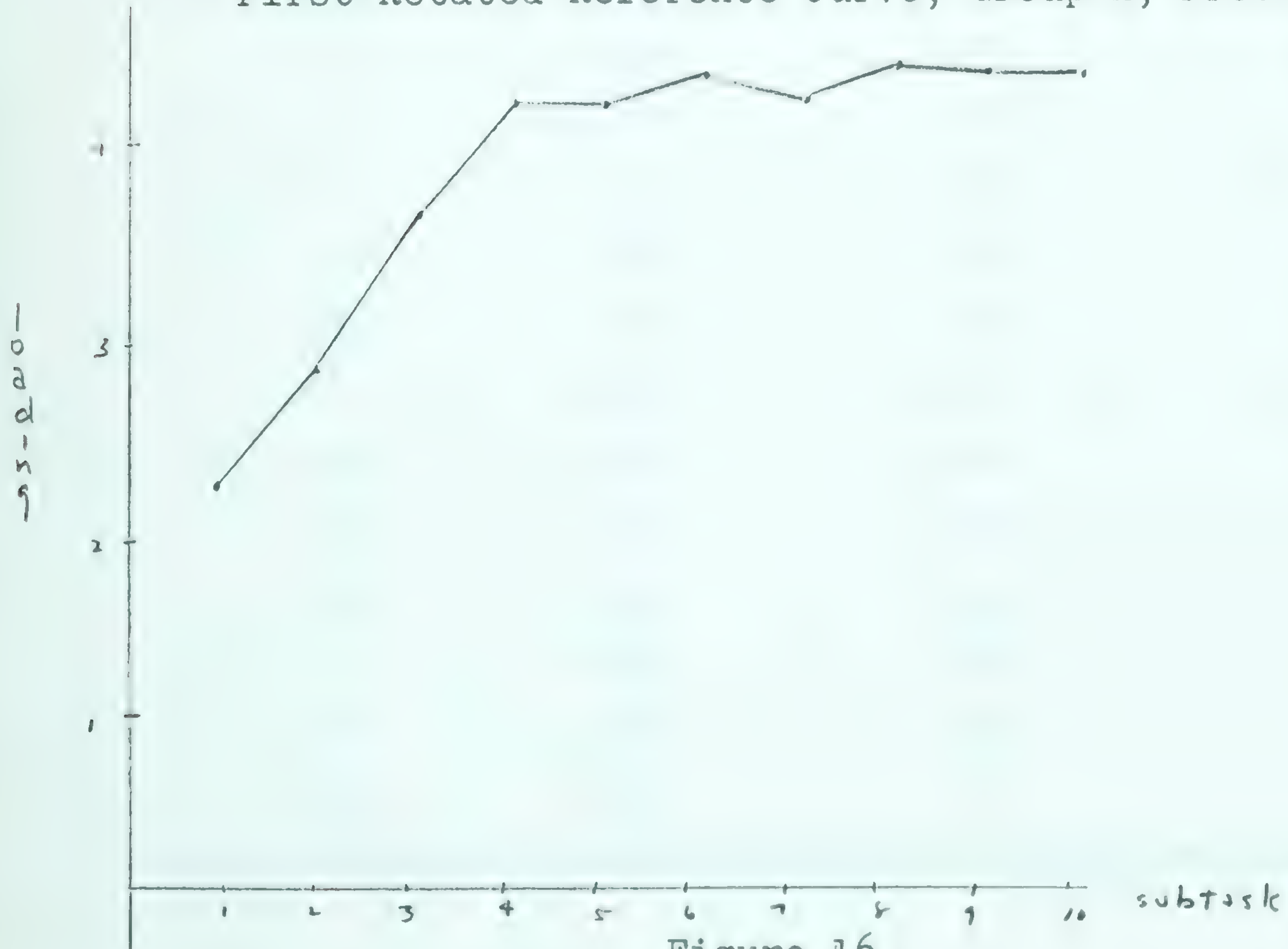


Figure 16

Second Rotated Reference Curve, Group A, Problem Two

TABLE VI

INDIVIDUAL LOADINGS ON ROTATED REFERENCE CURVES, GROUP A

PROBLEM TWO

Individual	I	II	Group Membership
1	.71	.80	2
2	1.45	.00	1
3	1.20	.24	1
4	1.20	.24	1
5	.96	.55	1
6	.55	.45	3
7	1.45	.00	1
8	.96	.55	1
9	.23	1.33	2
10	.26	1.26	2
11	1.20	.24	1
12	1.45	.00	1
13	1.20	.24	1
14	.21	1.26	2
15	.96	.55	1
16	.86	.73	1
17	-.25	1.73	2
18	-.26	1.68	2
19	1.58	1.09	3
20	-.91	2.46	2
21	1.16	.20	1
22	-.46	1.93	2
23	.96	.55	1

The value of T^2 was 50.24 whereas the value necessary for significance is only 14.41. Therefore it may be assumed that the two groups differ with respect to their loadings on the component curves.

A descriptive investigation of individuals who loaded highly on either curve was conducted to aid in the description of the component curves.

Selected Descriptions of Performance on Curve One

If the individual correctly identified the concept using the shape and shading portions of the answer sheet, and if he indicated that he was sure that he had attained the correct concept, then it was assumed that he had attained the concept on the card where these conditions were both met.

Subject 2, (loading 1.60) named the correct concept on the third card. This subject seemed to select a concept and maintain it until it was disproven.

Subject 5, (loading 1.29) named the correct concept on the eleventh card. This subject appeared to consider shapes and shades independently.

Subject 7, (loading 1.24) named the correct concept on the ninth card. He worked with shapes and shades independently. He hypothesized each attribute of the card and then put them together on the ninth card.

Subject 9, (loading 1.60) performed the same as subject 2.

Subject 10, (loading 1.53) hypothesized the concept on the fifth card but did not stick with it. He returned to it on the twelfth card.

Subject 19, (loading 1.60) did not solve the problem, although he hypothesized it several times (cards 3, 5, 24, 29). During the first three subtasks this seemed to be narrowing the concept down to the correct shading attribute but could not seem to connect it with the correct shape.

From these descriptions, curve one appears to be an efficient learning curve. Individuals who loaded highly on it, were individuals who attained the concept quickly. In addition, Curve one appears to be related to failure to attain the concept. All individuals in this category had fairly high (1.20, 1.03, .86, 1.60, .73) loadings on this curve. All these people seemed to be narrowing down the number of alternatives quite well but could not put the correct concept together. Therefore this curve is more accurately defined as an early efficiency curve.

Selected Descriptions of Performance on Curve Two

Subject 11, (loading 1.71) attained the concept at about the sixty-fifth exemplar. In the first subtask the subject appeared to have solved the shading part of the problem but could not connect this with the correct shape. Thereafter he seemed to be guessing.

Subject 13, (loading 2.36) hypothesized the correct solution on the eighth card but did not stick to the correct combination until the fortieth card.

Subject 15, (loading 1.63) correctly identified the concept at the sixtieth card. This subject seemed to be selecting several hypotheses simultaneously and trying to discard those that were disproven. Eventually he selected the correct shading attribute but did not connect it to the correct shape until later.

Subject 17, (loading 1.33) was very similar to subject 15.

Curve two appears to be characterized by less efficient learning. By less efficient it is meant that the less efficient individuals reject an hypothesis even though the exemplars are not infirming. Individuals in group two appear to have difficulty in putting shape attributes and shade attributes together.

In examining the rotated curves for problem one (figures 13 and 14), the first curve shows a rapid rise indicating early efficiency, then a slight drop-off toward the end. This latter characteristic is likely related to the high loadings of the individuals who did not solve the problem.

Curve two is characterized by very little change prior to the fifth subtask and a very rapid rise to the sixth subtask. This curve indicates less efficient

learning. The drop in the curve between subtasks two and five may arise as a result of individuals finding it necessary to overcome some set. On the other hand this may be a function of differential difficulty in the subtask, as no attempt was made to equalize the amount of new information in each subtask. As was seen by the description above many of the individuals who loaded highly on the second curve had such a difficulty inasmuch as they looked at only one attribute at a time.

The Relationship Between Groups, Personality Variables and Cognitive Variables

The table of intercorrelations among the individual loadings, the cognitive variables and the personality variables is given in Table VII. Since only a few subjects were involved in computing the correlations, a correlation of .40 was necessary to be significantly different from zero. The only significant correlation found between the predictor variables (personality and cognitive variables) and the loadings on the component curves, was that found between the individual loadings on the second curve and the controlled versus uncontrolled scale of the H.S.P.Q. This was a correlation of -.524. The correlation of the same scale with loadings on the first curve was almost significant (.324). Variable 13 or Q_3 as it is designated by the authors is described by the

authors (Cattell, Beloff, and Coan (1958, p. 37) as follows:

> The loaded response items show the child high on Q_3 as self-controlled, striving to accept approved ethical standards, ambitious to do well, considerate of others, foresighted, disposed to reduce and control expression of emotion, and conscientious.

Its negative correlation with loadings on the second curve could be explained in terms of the individual's desire to do well in the task. The subjects who loaded highly on the second curve may have been inclined to take the task less seriously than the subjects who loaded highly on the first curve. This could also be explained in terms of flexibility. Those loading highly on curve two being more flexible and perhaps searching for alternate solutions.

The generally low correlations that occurred between the predictors and loadings on curve one may be a result of high loadings being attained by those who solved the problem most efficiently as well as those who did not solve the problem.

Several variables were selected for further study. These variables were selected on the following bases, either relatively high absolute correlation with either of the curves, or low correlations with the curves but high correlation with none of the previously selected variables. The variables that were selected and the reasons for selection are given below.

TABLE VII[*]

INTERCORRELATIONS AMONG PREDICTORS AND INDIVIDUAL LOADINGS, GROUP A PROBLEM A (N=23)

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HSPQ A	.74	210	094	468	005	342	024	-292	361	309	-449	-107	-204	-407	519
HSPQ B		.75	-367	211	-076	-044	224	-221	101	-197	-112	013	-470	-288	342
HSPQ C			.77	-083	294	-011	-140	188	026	509	-232	-395	080	084	-014
HSPQ D				.78	098	380	018	036	250	223	066	264	-236	-235	509
HSPQ E					.72	-155	243	243	-075	-137	-085	-034	428	249	-187
HSPQ F						.68	-094	114	124	268	-003	038	-436	199	408
HSPQ G							.71	224	255	-255	-236	177	357	134	-063
HSPQ H								.80	-064	-128	-029	-187	141	395	066
HSPQ I									.79	-186	079	098	-096	006	200
HSPQ J										.69	-222	-056	-309	-315	232
HSPQ O											.72	432	-175	180	-152
HSPQ Q_2												.70	-172	-017	-067
HSPQ Q_3													.73	127	-486
HSPQ Q_4														.78	-344
H.C Verbal															.85

[*] Decimal Points have been omitted.

Reliabilities have been placed in the diagonal elements where possible.

TABLE VII (continued)

Variable	16	17	18	19	20	21	
HSPQ A	096	354	277	087	-115	087	1
HSPQ B	-225	-017	158	-114	-214	317	2
HSPQ C	334	-121	201	229	-036	-082	3
HSPQ D	040	028	198	093	109	052	4
HSPQ E	063	-404	-195	-269	098	-239	5
HSPQ F	376	167	-145	261	105	-041	6
HSPQ G	-157	-001	-044	-143	040	-147	7
HSPQ H	420	-045	040	341	181	-155	8
HSPQ I	-145	456	211	055	-064	032	9
HSPQ J	518	-052	321	333	074	-016	10
HSPQ O	-278	-030	-082	044	062	040	11
HSPQ Q_2	-224	-084	-118	114	114	054	12
HSPQ Q_3	-129	-184	-206	-389	324	-512	13
HSPQ Q_4	049	-338	-436	138	-005	-126	14
H.C. Verbal	288	566	584	298	086	026	15
H.C. Spatial	.86	040	078	376	227	-221	16
H.C. Numerical		.92	437	162	-084	173	17
H.C. Reasoning			.76	556	259	-189	18
Memory				___	294	-169	19
Curve I					___	-930	20
Curve II						___	21

Variable two was selected on the basis of a relatively high correlation with the second curve. This variable is the dull-bright scale of the H.S.P.Q.

Variable three was selected as a suppressor variable for variable two.

Variables six and fifteen were selected as suppressors for variable thirteen.

Variable nine was originally selected as a predictor variable but subsequent computations indicated that a clerical error had been made in the correlation figures resulting in variable nine not being important for any reason.

Variable thirteen was selected as a predictor variable.

Variables sixteen, eighteen, and nineteen showed some indications of being predictor variables on the basis of above average correlations with the curves, although none were significantly different from zero. It was felt that under conditions where the group three subjects were treated separately these variables might be more important.

Table VIII is a part of Table VII but is included for clarity to show the intercorrelations among the selected variables.

The canonical correlation coefficient was computed between the selected variables and the individual loadings on the component curves of the first problem. Calculations were made using the procedure outlined in Cooley and Lohnes (1962, p.36). The first canonical correlations was found to be .770.

Using procedures outlined in Bartlett (1941, 1947), the significance of the canonical correlation was determined. In this procedure λ is calculated using the formula 6, where λ is the canonical correlation squared, and i varies from one to the lesser of, the number of predictors or the number of criteria. $\Lambda = \prod_{i=1}^{q} (1 - \lambda_i)$ . . . (6)

The distribution of Λ is approximated by a χ^2 distribution where $\chi^2 = -(N - .5\ p + q + 1) \ln \Lambda$, with pq degrees of freedom.

The first canonical correlation was .770, and the second was .599. Using the above formula, Λ was found to be .2608, resulting in a χ^2 of 22.90. This was found to be significant only at the .20 level.

Because of the small number of subjects, and because of the problem mentioned previously of high loadings of those who did not solve the concept task, this dubiously significant correlation may provide indications for future study.

The weights assigned to the variables are given in Table IX.

The most important predictor variables appear to be variables 13, 6, and 18. Variable 13 has been described previously. The dimensions of variable 6 have been described by Cattell, Beloff and Coan (1958, p. 30) as:

Silent, Introspective vs. Talkative
Concerned, Brooding vs. Serene, Happy-go-lucky

TABLE VIII

INTERCORRELATIONS AMONG SELECTED PREDICTORS AND INDIVIDUAL LOADINGS*

(N = 23)

	Variable										
	2	3	6	9	13	15	16	18	19	20	21
HSPQ B		-367	-044	101	-470	342	-225	158	114	214	317
HSPQ C			-011	026	080	-014	334	201	229	-036	-082
HSPQ F				124	-436	408	376	-145	261	105	-041
HSPQ I					-096	200	-145	211	055	-064	032
HSPQ Q_3						-486	-129	-206	-389	324	-512
HC Verbal							288	584	298	086	026
HC Spatial								078	376	227	-221
HC Reasoning									556	259	-189
Memory										294	-169
Curve I											-930
Curve II											___

---

*Decimals have been omitted.

TABLE IX

VARIABLE WEIGHTS FOR MAXIMUM CANONICAL CORRELATION PREDICTORS

Variable	Weight
2	-.107
3	-.009
6	-.398
9	-.002
13	-.783
15	.213
16	-.160
18	-.383
19	-.020
	Criteria
I	.472
II	.882

Depressed vs. Cheerful

Incommunicative, Sticks to Inner Values vs. Frank, Expressive Mercurial

Languid, Slow vs. Quick and Alert

This variable seems to be related to an introversion-extraversion dimension.

Variable 18 is the Reasoning Scale of the Holzinger-Crowder Unifactor Test.

Because these variables have negative weights, and because the second component curve is weighted more heavily than the first, one is left with the tentative conclusion that the less efficient problem solvers tend to be more introverted, less conscientious, and the poorer reasoners than the more efficient problem solvers. It must be kept in mind, however, that these conclusions can be treated only as indications.

To overcome the difficulty of the subjects who did not solve the problem a discriminant function was calculated to determine what variables were important in distinguishing between these groups. Unfortunately, the number of subjects in each group is too small to draw any absolute conclusions.

A stepwise procedure was used in which the variable which contributed least to the discriminant function was removed at that step. In this way the multiple correlation between the predictors and the function was calculated,

and the significance tested. Table X shows the coefficients of each variable in the function as well as the multiple correlation and the level of significance. Table XVIII in Appendix G shows the variance-covariance matrix. For further analysis the variables involved at the sixth step were used. This decision was made in an effort to bring about a compromise between high correlation on one hand and as few as possible predictors on the other.

The variables selected were 6, 13, 18, and 19. Three of these have been described previously and were of importance in the canonical correlation. The fourth, the memory test, is described in detail in chapter three.

The means of the variables for each group are given in Table XI. It can be seen that the means of Group one are higher than the means of Group two in all areas except variables two and three.

The difference between the two groups on the basis of the four selected variables was tested using Hotelling's T^2 test. The observed value of T^2 was 6.54. The value of T^2 required for significance at the .05 level is 15.65. It can therefore be concluded that the two groups can not be separated with any degree of accuracy.

Results: Transfer

In attempting to determine the role of transfer in this study, two aspects seem to be important. The first important factor is the shape of the component curve, and

TABLE X

TABLE OF COEFFICIENTS AND MULTIPLE CORRELATIONS AT EACH STEP OF THE DISCRIMINANT FUNCTION

Step	Variable Coefficients										sig.
	2	3	6	9	13	15	16	18	19	R	
1	.607	-.058	.134	.040	.282	-.006	.006	.015	.052	.605	n.s.
2	.066	-.051	.129	.035	.294	-	.005	.012	.057	.603	n.s.
3	.066	-.049	.136	-	.299	-	.004	.013	.055	.593	n.s.
4	.091	-	.156	-	.328	-	.003	.014	.055	.580	n.s.
5	.083	-	.173	-	.325	-	-	.015	.057	.571	.01
6	-	-	.157	-	.273	-	-	.017	.044	.541	.01
7	-	-	.200	-	.274	-	-	.029	-	.477	.05
8	-	-	.116	-	-	-	-	.020	-	.237	n.s.
9	-	-	-	-	-	-	-	.014	-	.098	n.s.

TABLE XI

MEANS OF VARIABLES SELECTED FOR DISCRIMINATION FUNCTION

Variable	Group One Mean	Group Two Mean
~~1~~2	7.10	7.63
~~2~~3	2.40	2.63
~~3~~6	4.70	3.88
9	2.90	2.63
13	2.40	2.00
15	37.40	34.50
16	65.70	49.75
18	46.90	39.88
19	15.00	10.88

the second is the membership of the groups defined by the component curves.

If transfer occurs one would expect that the shapes of the curves in problem two would tend to have greater initial acceleration than the corresponding curves in problem one. The reason for this being, that in the second task the individuals would neither have to become familiar with the task nor would they have to search for ways to solve it. Furthermore, transfer should be indicated by change in group membership, that is, a change from high loadings on less-efficient curves to high loadings on more efficient curves. In either case the transfer should result in an increase in efficiency of problem solution.

In problem one the first rotated reference curve (figure 9) reaches a peak on the third subtask, whereas in problem two (figure 11) the curve attains a peak in the second subtask. The second curve (figure 10) in problem one does not begin to rise until the fifth subtask, but the corresponding curve (figure 12) in problem two accelerates from the first to the fourth subtask. In problem one the first curve undergoes fluctuation in the last five subtasks whereas the first curve in the second subtask is steadier. This seems to indicate that the curve shapes in problem two are representing more efficient learning processes.

Hunka (1961) suggests a technique for investigating

TABLE XII

INTERCORRELATIONS BETWEEN INDIVIDUAL LOADINGS ON THE TWO COMPONENT CURVES FOR PROBLEMS ONE AND TWO OF GROUP A

	Problem One		Problem Two	
	I	II	I	II
I	1.000			
II	-.929	1.000		
I	.150	-.211	1.000	
II	-.219	.353	-.946	1.000

the role of transfer by calculating the intercorrelations between factor loadings for both problems. Table XII gives these intercorrelations.

Table XII indicates that the loadings on the second component curves for each problem have a low positive correlation (significantly different from zero at the .10 level). This as well as the non-significant correlation for the first curves would indicate that the loadings of the subjects are changing from the first problem to the second. With this in mind the subjects were investigated on the basis of the groups defined in Tables V and VI.

Table XIII indicates the group membership in the problems. Transfer would be indicated by a change in group membership from a less efficient curve to a more efficient curve, that is from group three to groups one or two, or from group two to group one. Transfer would be also indicated by maintaining membership in either group two or group one, because the curves themselves indicate increase in efficiency.

Using this as a criterion for transfer seventeen of the twenty-three subjects indicated that transfer had occurred.

In looking at the original data to check the validity of this conclusion, it is noted that in problem two eighteen subjects had perfect subtask scores as early as the fourth subtask twelve of whom had perfect scores

as early as the second subtask. The corresponding figures for problem one were eight and three. This seems to indicate that either the concept task for problem two was easier than for problem one or that a "learning how to learn" effect had occurred. The former conclusion is discarded on the basis of research in the area of concept difficulty, wherein difficulty was found to be a function of the number of relevant attributes, implying that the tasks were of equal difficulty.

Results: Task Difficulty

Group B was used exclusively to investigate the relationship between difficulty and complexity. The results are inconclusive.

In problem one, nineteen of twenty-seven subjects solved the concept problem, whereas in problem two only two attained the concept neither of whom solved problem one.

It was hypothesized that the component curves for problem two would be more complex than for problem one either from the point of view of number or of shape. It was found that only one curve was significant for each problem. Some slight support is given the hypothesis on examination of Figures 7 and 9. The curve of the second problem (figure 9) appears to have more pronounced irregularities than that of the first problem. The general shape in both cases appears to be very similar.

TABLE XIII

NUMBER OF SUBJECTS IN GROUP A DISTRIBUTED ACCORDING TO GROUP MEMBERSHIP IN PROBLEM ONE AND PROBLEM TWO

		Problem Two			
		Group			
		1	2	3	Total
Problem	1	8	3	0	11
One	2	2	4	1	7
	3	2	1	2	5
Total		13	7	3	23

The problem with this portion of the study lies in the fact that the second problem was too difficult for the subjects. Therefore the results must be classified as inconclusive.

CHAPTER VI

SUMMARY OF FINDINGS AND IMPLICATIONS FOR FUTURE RESEARCH

Matrices of sums of squares and cross-products resulting from Brunerian type concept tasks were analyzed into component curves using the method developed by Tucker (1960). On one task the individuals were grouped on the basis of their factor loadings. The relationship between these loadings and selected cognitive and personality variables was investigated.

Results of a second concept task were compared to the first task to see if transfer of training occurred.

An investigation was conducted into the relationship between task difficulty and component curve complexity.

Results

Hypothesis one: It was found that individuals could be divided into groups according to their loadings on component curves. The success of this classification is dependent on the tasks being of optimum difficulty. In the present study the groups could be described as more efficient and less efficient.

Hypothesis two: It was found that the groups thus classified did not differ significantly on the variables selected. It was thought that this may be partially a result of the small sample. Within the limits of the study

three variables seemed important. The more efficient problem solvers seemed to be more anxious to do well, they seemed to be better reasoners, and they seemed to be somewhat more extroverted than the less efficient problem solvers.

Hypothesis three: The component curves indicated a learning how to learn function inasmuch as curves on the second problem reached their maxima much earlier than curves on the first problem. This was verified by comparison with the score matrices.

Hypothesis four: The relationship between task difficulty and curve complexity was not determined although there were indications that as the task became more difficult the curves became more complex.

IMPLICATIONS FOR FURTHER RESEARCH

General Implications

The importance of this study has been in its demonstration of the application of component curve analysis and multivariate techniques to concept formation. Inasmuch as the small sample made the results inconclusive, replication is in order.

Certain of the variables selected for investigation such as many of the HSPQ variables might very well be dropped, and better measures of others used. Such variables as introversion-extraversion and attitudes towards school may be important. In any event an effort should

be made to use personality tests specifically designed to test one aspect. Reasoning tests should be considered in finer divisions to discover the role played by reasoning.

The concept task itself should be analyzed in terms of information theory to ensure the standardization of difficulty. The amount of new information presented with each stimulus should be made known making it possible to analyze concept formation and problem solving in terms of information used as well as information presented. The score for an individual might be expressed in terms of information used per unit amount of information given. The component curves thus derived may be more useful than those obtained from a simple score matrix.

Implications with Respect to the Use of the Technique

This study was, in part, an attempt to adapt a new technique to a new problem. The component curve technique of analyzing the learning process wouls seem to be particularly applicable in learning situations consisting of discrete learning experiences. An example of such a situation may arise in the learning of a motor skill.

The technique would also seem important in situations where the classification of individuals on the basis of different ways of learning is necessary. Because the classification is determined by the rotation of the component curves, a recent paper by Cliff (1962) discussing the analytic rotation of factors to a functional relationship, may be important.

BIBLIOGRAPHY

BIBLIOGRAPHY

Baggaley, A. R. Concept formation and its relation to cognitive variables. *J. gen. Psychol.* 1955, *52*, 297-306.

Bartlett, M. S. The statistical significance of canonical correlations. *Biometrika.* 1941, *32*,29-38.

Bartlett, M. S. Multivariate analysis. *Supplement to the Journal of the Royal Statistical Society.* 1947, *9*, 176-179.

Blatt, S. J. and Stein, M. I. Efficiency in problem solving. *J. Psychol.* 1959, *48*, 193-213.

Bricker, P. D. The identification of redundant stimulus patterns. *J. exp. Psychol.* 1955, *49*, 73-81.

Bruner, J. S. Goodnow, J. J. and Austin, S. A. *A study of thinking.* New York, John Wiley and Sons, 1956.

Buss, A. H. A study of concept formation as a function of reinforcement and stimulus generalization. *J. exp. Psychol.* 1950, *40*, 494-503.

Cahill, H. E. and Hovland, C. I. The role of memory in the acquisition of concepts. *J. exp. Psychol.* 1960, *59*, 137-144.

Catell, R. B., Beloff, H. and Coan, R. W. *Handbook for the IPAT High School Personality Questionair.* Champaign, Ill., Institute for Personality and Ability Testing. 1958.

Cliff, N. Analytic rotation to a functional relationship. *Psychometrika.* 1962, *27*, 283-296.

Cooley , W. W. and Lohnes, P. R. *Multivariate procedures for the behavioral sciences.* New York, John Wiley and Sons. 1962.

Duncan, C. P. Recent research on human problem solving. *Psychol. Bull.* 1959, *56*, 397-429.

Eckart, C., and Young, G. The approximation of one matrix by another of lower rank. *Psychometrika.* 1963, *1*, 211-218.

Eckstrand, G. A. and Wickens, D. O. Transfer of perceptual set. J. exp. Psychol., 1954, 47, 274-278.

English, H. B. An experimental study of certain initial phases of the process of abstraction. Amer. J. Psychol., 1922, 33, 305-350, cited by Vinacke, W. E., The Psychology of Thinking, New York, McGraw-Hill, 1952.

Estes, W. K. The problem of inference from curves based on group data. Psychol. Bull., 1956, 53 134-140.

Eysinck, H. J. Personality and problem solving. Psychol. Rep. 1959, 5, 592.

Fillenbaum, S. and Jackman, A. Dogmatism and anxiety in relation to problem solving: An extension of Rokeach's results. J. Abnorm. Soc. Psychol. 1961, 63, 212-214.

Fisher, S. C. The process of generalization and abstraction and its product, the general concept. Psychol. Monogr. 1916, No. 90, as cited in Vinacke, W. E. A Study of thinking, New York, McGraw-Hill, 1952.

Goldstein, K. and Sheerer, M. Abstract and concrete behavior: An experimental study with special tests. Psychol. Monogr. 1941, 53, (2, whole No. 239).

Guilford, J. P. Factors in problem solving. A.R.T.C. Instructor J. 1954, 4, 197-204, as cited in Harootunian, B. and Tate, M. W. The relationship of certain selected variables to problem solving ability. J. educ. Psychol. 1960, 51, 326-333.

Hanfman, E. and Kasanin, J. A method for the study of concept formation. J. Psychol. 1937, 3, 521-540.

Harootunian, B. and Tate, M. W. The relationship of certain selected variables to problem solving ability. J. educ. Psychol. 1960, 51, 326-333.

Hayes, K. J. The backward curve: A method for the study of learning. Psychol. Rev. 1953, 60, 269-275.

Hilgard, E. R. A summary and evaluation of alternative procedures for the construction of Vincent curves. Psych. Bull. 1938, 35, 282-297.

Holzinger, K. J. and Crowder, N. A. Holzinger-Crowder Uni-Factor Tests. Manual. New York: World Book Co., 1955.

Hunka, S. M. A study of generalized learning curves. Unpublished doctoral dissertation, Univer. of Illinois, 1961.

Maltzman, I., Eisman, E. and Brooks, L. O. Some relationships between methods of instruction, personality variables and problem solving behavior. J. educ. Psychol. 1956, 47, 71-78.

McNemar, O. W. An attempt to differentiate between individuals with high and low reasoning ability. Amer. J. Psychol. 1955, 68, 20-36.

Moore, T. The process of abstraction, Univ. of Calif. Publ. Psychol. 1910, 1, 73-197. As cited in Vinacke, W. E., A study of thinking. New York, McGraw-Hill, 1952.

Neal, A. F. The relationship of authoritarian personality to learning. J. educ. Psychol. 1959, 50, 195-199.

Olsen, L. A. Concept attainment of high school sophomores. J. educ. Psychol. 1963, 54, 213-216.

Osler, S. F. and Fivel, M. W. Concept attainment I: The role of age and intelligence in concept attainment by induction. J. exp. Psychol. 1961, 62, 1-8.

Osler, S. F. and Trautman, G. E. Concept attainment II: Effect of stimulus complexity upon attainment at two levels of intelligence. J. exp. Psychol. 1961, 62, 9-13.

Osler, S. F. and Weiss, S. R. Concept attainment III: Effect of instruction at two levels of intelligence. J. exp. Psychol. 1962, 63, 528-533.

Reed, H. B. The learning and retention of concepts IV: The influence of complexity of stimuli. J. exp. Psychol. 1946, 36, 252-261.

Rhine, R. J. The effect on problem-solving of success or as a function of cue specificity. *J. exp. Psychol*. 1957, *53*, 121-125.

Romanow, C. V. Anxiety level and ego involvement as factors in concept formation. *J. exp. Psychol*. 1958, *56*, 166-173.

Seigal, S. *Nonparametric statistics for the behavioral sciences*. New York, McGraw-Hill, 1956.

Shepherd, R. N., Hovland, C. I. and Jenkins, A. M. Learning and memorization of classifications. *Psychol. Monogr*. 1961, *75* No. 13 (Whole No. 517)

Shore, E. and Sechrest, L. Concept attainment as a function of the number of positive instances presented. *J. educ. Psychol*. 1961, *52*, 303-307.

Sidman, M. A note on functional relations obtained from group data. *Psychol. Bull*., 1952, *49*, 263-269.

Smoke, K. L. Negative instances in concept learning. *J. exp. Psychol*., 1933, *16*, 583-88.

Staats, A. W. Verbal and instrumental response hierarchies and their relationship to problem solving. *Amer. J. Psychol*. 1957, *70*, 442-446.

Stewart, E. E. A factorial study of concept learning. *Dissertation Abstracts*, 1960, *20*, 2895.

Tucker, L. R. Determination of parameters of a functional relationship by factor analysis. *Psychometrika*, 1958, *23*, 19-24.

Tucker, L. R. Determination of generalized learning curves by factor analysis. *Technical report*. Princeton, Educational Testing Service. 1960.

Vinacke, W. E. *A study of thinking*, New York, McGraw-Hill, 1952.

Wallach, L. The complexity of concept attainment. Amer. J. Psychol. 1962, 75, 277-283.

Weitzman, R. A. A factor analytic method for investigating differences between groups of individual learning curves. Psychometrika, 1963, 28, 1-9.

Wesley, E. Perservative behavior in a concept formation task as a function of manifest anxiety and rigidity. J. Abnorm. soc. Psychol. 1953, 48, 129-134.

Winer, B. J. Statistical Principles in Experimental Design. New York, McGraw-Hill, 1962.

Yudin, L. and Kates, S. L. Concept attainment and adolescent development. J. educ. Psychol. 1963, 54, 177-182.

APPENDIX A

APPENDIX A

Instructions for Tasks

I am going to show you some cards. On these cards are triangles, circles, and squares (draw on blackboard and show some examples). Some of these shapes are white, some are black and some are shaded (draw on blackboard and show examples).

Some of the cards that I will show you are "yes" cards and some are "no" cards. I want you to find out which are yes cards. An example of a yes card might be any card with a black figure, or all cards with a circle.

Notice that the position of the figure on the card is not important. The only important things are shape and color.

Look, now, at the answer sheet. You will notice that there are spaces for eighty answers. What we shall do is this: I shall show you a card which is a yes card, then I shall show you another card. If you think that it is a yes card, put an X in the box under YES for question one. If you think it is a no card, put an X in the box under NO for question one. Next to the YES and NO boxes you will see a triangle, a circle, and a square, also a black shape, a shaded shape, and a white shape. If you think that yes cards have something to do with triangles put a stroke through the triangle. If you think that yes cards have something to do with shaded squares put a stroke

through the shaded shape and through the square. If you can't decide whether yes cards have something to do with squares or circles, put a stroke through both.

Disregard the numbers 1, 2, 3.

Finally, there is a line at the top of the page on the top line. It says DON'T KNOW on the left, QUITE SURE in the middle and VERY SURE on the right. This is to show me how sure you are of what makes up a "yes" card. If you don't have any idea then put a small x on the line at the left end. As you catch on to what makes a "yes" card you should put the x nearer the centre of the line. When you are sure that you have the correct answer then put it on the right side.

After you have marked in your answer for a particular card then I shall give you the correct answer. Once you are absolutely sure that you have the correct answer then you need not mark on the shapes, shades, or line anymore. If, however, you decide later on that you did not have the correct answer then you should start marking the answer sheet again. For every card you must make a mark in the YES box or the NO box.

Are there any questions? (Answer questions and re-explain if necessary.)

Here is an example to do. You will have about fifteen seconds to do each card.

APPENDIX B

APPENDIX B

THE POPULATION OF CARDS

SHAPE

Number	Circle	Triangle	Square
1	W*	W	W
2	W	W	S
3	W	S	W
4	S	W	W
5	W	S	S
6	S	W	S
7	S	S	W
8	W	B	S
9	S	W	B
10	B	W	S
11	W	S	B
12	S	B	W
13	B	S	W
14	W	W	B
15	W	B	W
16	B	W	W
17	B	B	W
18	B	W	B
19	W	B	B
20	S	S	S
21	S	S	B
22	S	B	S
23	B	S	S
24	B	B	S
25	B	S	B
26	S	B	B
27	B	B	B
	Triangle	Triangle	Triangle
28	W	W	W
29	W	W	S
30	S	S	W
31	W	S	B
32	B	W	W
33	B	B	W
34	S	S	S
35	B	S	S
36	S	B	B
37	B	B	B

Number	Square	Square	Square
38	W	W	W
39	W	W	S
40	S	S	W
41	W	S	B
42	B	B	W
43	B	W	W
44	S	S	S
45	B	S	S
46	S	B	B
47	B	B	B
	Circle	Circle	Circle
48	W	W	W
49	W	W	S
50	S	S	W
51	W	S	B
52	B	W	W
53	B	B	W
54	S	S	S
55	B	S	S
56	B	B	S
57	B	B	B
	Circle	Triangle	Triangle
58	W	W	W
59	S	W	W
60	W	S	W
61	W	S	S
62	S	S	W
63	B	S	W
64	S	B	W
65	W	S	B
66	B	W	W
67	W	B	W
68	B	B	W
69	W	B	B
70	S	S	S
71	B	S	S
72	S	S	B
73	S	B	B
74	B	B	S
75	B	B	B

*W stands for white

B stands for black

S stands for striped

	Square	Triangle	Triangle
76	W	W	W
77	S	W	W
78	W	S	W
79	S	S	W
80	W	S	S
81	B	S	W
82	S	W	W
83	W	B	S
84	B	W	W
85	W	B	W
86	B	B	W
87	W	B	B
88	S	S	S
89	B	S	S
90	S	B	S
91	S	B	B
92	B	S	B
93	B	B	B

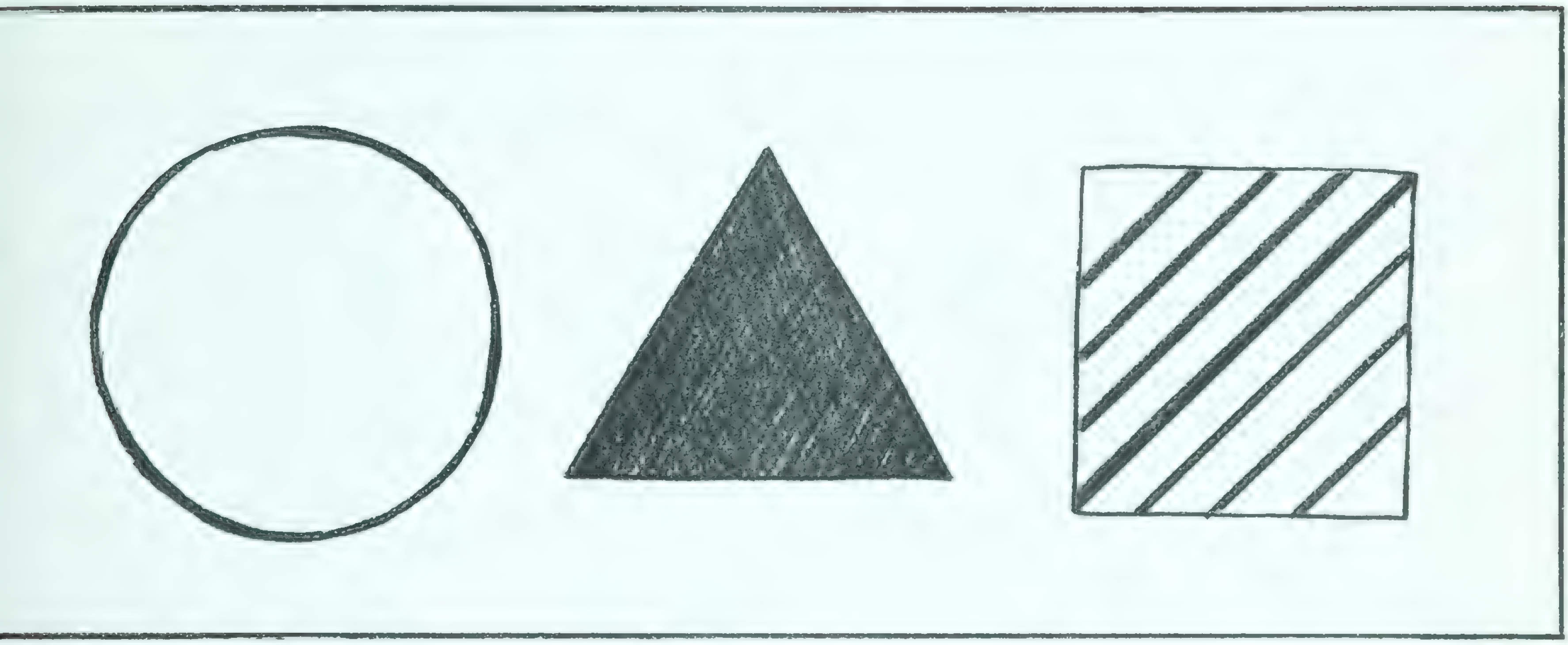
	Triangle	Circle	Circle
94	W	W	W
95	S	W	W
96	W	S	W
97	W	S	S
98	S	S	W
99	B	S	W
100	S	B	W
101	W	S	B
102	B	W	W
103	W	B	W
104	B	B	B
105	W	B	B
106	S	S	S
107	B	S	S
108	S	B	S
109	S	B	B
110	B	B	S
111	B	B	B

	TRIANGLE	SQUARE	SQUARE
112	W	W	W
113	S	W	W
114	W	S	W
115	B	S	W
116	W	S	S
117	B	S	W
118	S	B	W
119	W	S	B
120	B	W	W
121	W	B	W
122	B	B	W
123	W	B	B
124	S	S	S
125	W	S	S
126	S	S	B
127	B	B	S
128	S	B	B
129	B	B	B

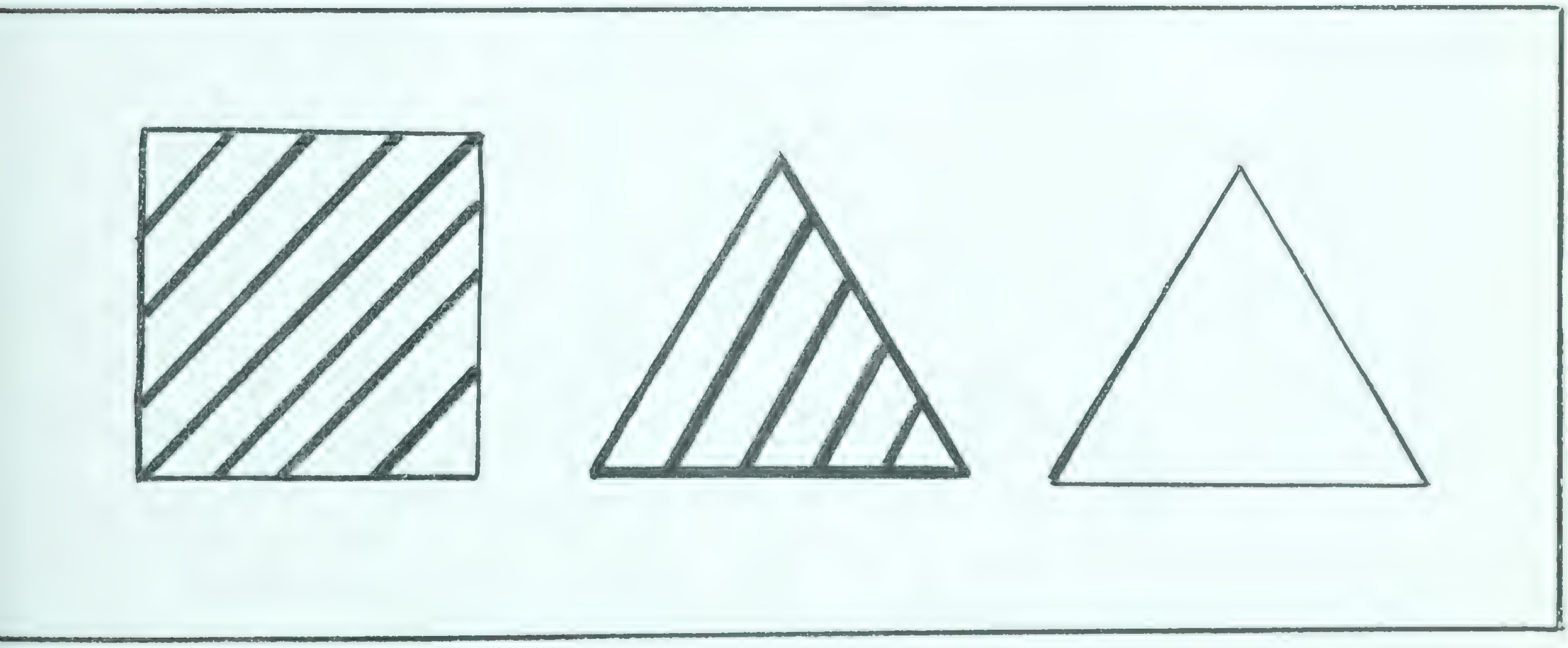
	Square	Circle	Circle
130	W	W	W
131	S	W	W
132	W	S	W
133	S	S	W
134	W	S	S
135	B	S	W
136	S	B	W
137	W	S	B
138	B	W	W
139	W	B	W
140	W	B	B
141	B	B	W
142	S	S	S
143	W	S	S
144	S	B	S
145	S	B	B
146	B	B	S
147	B	B	B

	Circle	Square	Square
148	W	W	W
149	S	W	W
150	W	S	W
151	W	S	S
152	S	S	W
153	B	S	W
154	S	B	W
155	W	B	S
156	B	W	W
157	W	B	W
158	W	B	B
159	B	B	W
160	S	S	S
161	B	S	S
162	S	B	S
163	S	B	B
164	B	S	B
165	B	B	B

EXAMPLES OF CARDS



CARD NUMBER 8



CARD NUMBER 79

[illegible]

[illegible]

[illegible]

APPENDIX C

APPENDIX C

THE TASKS

The card numbers are the numbers defined in the population of Appendix B.

Example Problem - A Triangle

Example 39

1	158	7	14*	13	17*
2	76*	8	149	14	140
3	161	9	163	15	143
4	162	10	164	16	153
5	125*	11	18*	17	64*
6	142	12	24*	18	104

Task One, A Shaded Square - Example #8

1	38	21	53	41	24*	61	19
2	138	22	12	42	15	62	32
3	10*	23	135	43	40 *	63	77*
4	123	24	131*	44	147	64	2*
5	121*	25	63	45	99	65	154
6	27	26	73	46	54	66	150*
7	61	27	76	47	47	67	98
8	95	28	109	48	30	68	11
9	133*	29	6*	49	125*	69	46*
10	50	30	87	50	42	70	130
11	152*	31	17	51	97	71	70
12	48	32	153*	52	93	72	110
13	112	33	134	53	44*	73	155*
14	7	34	94	54	102	74	79*
15	52	35	143	55	105	75	3
16	84	36	88*	56	43	76	14
17	67	37	74	57	159	77	35
18	108	38	13	58	21	78	81
19	114*	39	18	59	25	79	80
20	68	40	45*	60	69	80	56

* Indicates that the card is an exemplar.

Task 2, A circle and a Square - Example #16

1	125	21	91	41	90	61	59
2	34	22	93	42	9*	62	37
3	165*	23	131*	43	116	63	115
4	100	24	117	44	84	64	13*
5	121	25	119	45	75	65	46
6	40	26	81	46	64	66	47
7	15*	27	83	47	118	67	60
8	43	28	144*	48	134*	68	160*
9	35	29	107	49	57	69	137*
10	73	30	4*	50	157*	70	70
11	69	31	39	51	87	71	31
12	36	32	111	52	129	72	102
13	49	33	104	53	66	73	113
14	8*	34	55	54	85	74	19*
15	62	35	128	55	141*	75	126
16	22*	36	110	56	96	76	32
17	92	37	72	57	56	77	17*
18	41	38	5*	58	124	78	101
19	117	39	23*	59	20*	79	33
20	3*	40	120	60	99	80	77

Task 3, A black triangle - Example #86

1	106	21	59	41	17*	61	87*
2	89	22	32*	42	16	62	130
3	145	23	64*	43	60	63	109
4	153	24	53	44	25	64	63
5	36*	25	66	45	84	65	22*
6	159	26	115	46	143	66	140
7	19*	27	38	47	56	67	92*
8	128	28	74 *	48	27*	68	123
9	156	29	126	49	79	69	136
10	47	30	119	50	129*	70	71
11	148	31	9	51	146	71	51
12	20	32	65*	52	78	72	54
13	73*	33	127*	53	50	73	58
14	14	34	23	54	72*	74	21
15	45	35	98	55	10	75	99*
16	69*	36	26*	56	101	76	163
17	139	37	103	57	82*	77	122*
18	94	38	18	58	121	78	81
19	1	39	137	59	29	79	147
20	95	40	28	60	151	80	7

*Indicates that the card is an exemplar.

APPENDIX D

NAME ______________________________

	Yes	No				Don't Know	Quite Sure	Very Sure
1.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
2.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
3.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
4.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
5.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
6.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
7.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
8.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
9.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
0.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
1.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
2.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
3.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
4.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			

	Yes	No				Don't Know	Quite Sure	Very Sure
15.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
16.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
17.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
18.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
19.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
20.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
21.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
22.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
23.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
24.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
25.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
26.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
27.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			
28.	☐	☐	△ ○ ☐	⬬ ▨ ⬭	1 2 3			

	Yes	No			Don't Know	Quite Sure	Very Sure		Yes	No			Don't Know	Quite Sure	Very Sure
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	43.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	44.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	45.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	46.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	47.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	48.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	49.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	50	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	51.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	52.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	53.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	54.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	55.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤
.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤	56.	☐	☐	△ ○ □	● 1 ▨ 2 ⬭ 3	├	┼	┤

	Yes	No		Don't Know	Quite Sure	Very Sure
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
.	☐	☐	△ ⬬ 1 ▨ 2 ⬭ 3	├	┼	┤

	Yes	No		Don't Know	Quite Sure	Very Sure
69.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
70.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
71.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
72.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
73.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
74.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
75.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
76.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
77.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
78.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
79.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤
80.	☐	☐	△ ⬬ 1 ○ ▨ 2 □ ⬭ 3	├	┼	┤

APPENDIX E

TABLE XIV

SCORE MATRIX, GROUP A, PROBLEM ONE

	Subtask									
Individual	1	2	3	4	5	6	7	8	9	10
1	4	7	8	8	8	8	8	8	8	8
2	7	8	8	8	8	8	8	8	8	8
3	4	4	5	8	8	8	8	8	8	8
4	4	5	7	5	4	4	6	5	7	2
5	4	6	8	8	8	8	8	8	8	8
6	4	4	2	7	3	2	6	3	2	5
7	2	8	7	8	8	8	8	8	8	8
8	5	8	5	6	4	5	8	8	8	8
9	7	8	8	8	8	8	8	8	8	8
10	7	8	8	8	8	8	9	8	8	8
11	6	5	4	2	2	6	6	8	8	8
12	5	7	4	5	5	6	7	5	4	7
13	4	4	4	2	2	8	8	8	8	8
14	4	6	5	6	6	3	6	8	8	8
15	4	1	5	4	6	8	8	8	8	8
16	2	5	3	4	3	8	8	8	8	8
17	3	8	3	5	5	8	8	8	8	8
18	4	3	4	5	5	2	6	6	8	8
19	4	4	6	6	5	1	3	3	4	2
20	4	3	4	8	7	8	8	8	8	8
21	2	5	4	5	7	7	7	7	5	5
22	2	4	4	5	7	8	8	8	8	8
23	3	4	5	8	8	8	8	8	8	8

TABLE XV

SCORE MATRIX, GROUP A, PROBLEM TWO

		Subtask									
		1	2	3	4	5	6	7	8	9	10
	1	7	6	7	8	8	8	8	8	8	8
	2	8	8	8	8	8	8	8	8	8	8
	3	7	8	8	8	8	8	8	8	8	8
	4	7	8	8	8	8	8	8	8	8	8
	5	6	8	8	8	8	8	8	8	8	8
	6	5	4	5	5	3	5	6	8	5	5
	7	8	8	8	8	8	8	8	8	8	8
	8	6	8	8	8	8	8	8	8	8	8
	9	3	8	8	8	8	8	8	8	8	8
	10	5	6	8	8	8	8	8	8	8	8
Individual	11	7	8	8	8	8	8	8	8	8	8
	12	8	8	8	8	8	8	8	8	8	8
	13	7	8	8	8	8	8	8	8	8	8
	14	3	8	8	8	8	8	8	8	8	8
	15	6	8	8	8	8	8	8	8	8	8
	16	6	8	7	8	8	8	8	8	8	8
	17	4	5	7	7	8	8	8	8	8	8
	18	4	4	5	6	5	7	8	8	8	8
	19	7	3	3	2	2	2	5	3	3	4
	20	3	3	8	8	8	8	8	8	8	8
	21	7	8	7	8	8	8	8	8	8	8
	22	6	3	3	8	8	8	8	8	8	8
	23	6	8	8	8	8	8	8	8	8	8

TABLE XVI

SCORE MATRIX, GROUP B, PROBLEM ONE

		Subtask									
		1	2	3	4	5	6	7	8	9	10
	1	2	3	5	3	3	4	7	7	5	5
	2	5	8	8	8	8	8	8	8	8	8
	3	4	7	7	8	8	8	8	8	8	8
	4	3	3	5	3	5	5	7	4	5	5
	5	4	7	4	8	7	8	8	8	8	8
	6	7	3	5	7	5	2	7	6	8	8
	7	4	7	8	8	8	8	8	8	8	8
	8	6	5	7	4	7	8	6	5	6	6
	9	4	8	8	8	8	8	8	8	8	8
	10	4	8	5	8	8	8	8	8	8	8
Individual	11	3	2	2	8	4	4	6	5	3	4
	12	4	8	8	8	8	8	8	8	8	8
	13	7	6	4	7	6	7	8	8	8	8
	14	4	8	8	8	8	8	8	8	8	8
	15	2	6	8	8	8	8	8	8	8	8
	16	4	8	8	8	8	8	8	8	8	8
	17	4	8	8	8	8	8	8	8	8	8
	18	4	4	5	7	3	4	1	3	4	4
	19	3	8	8	8	8	8	8	8	8	8
	20	5	6	8	8	8	8	8	8	8	8
	21	2	7	8	8	8	8	8	8	8	8
	22	3	8	6	7	6	5	6	7	5	5
	23	4	4	8	8	8	8	8	8	8	8
	24	2	3	3	4	3	3	6	6	2	6
	25	5	8	8	8	8	8	8	8	8	8
	26	4	7	8	8	8	8	8	8	8	8
	27	4	3	3	3	6	4	3	4	2	3

TABLE XVII

SCORE MATRIX, GROUP B, PROBLEM TWO

		Subtask									
		1	2	3	4	5	6	7	8	9	10
	1	5	7	5	6	6	6	7	6	6	7
	2	7	8	8	8	8	6	5	8	7	7
	3	5	6	5	4	4	6	7	6	7	5
	4	6	4	3	5	5	4	7	4	4	5
	5	2	5	4	4	6	4	7	5	4	4
	6	5	7	5	7	5	5	7	6	7	7
	7	5	5	4	7	7	5	5	3	3	7
	8	6	6	6	7	7	4	6	6	6	7
	9	4	5	5	3	5	2	5	4	2	3
	10	5	4	6	7	6	7	7	3	6	6
Individual	11	5	4	4	8	8	8	8	8	8	8
	12	5	5	6	4	8	3	6	4	4	5
	13	5	5	5	7	5	7	6	8	7	6
	14	7	6	5	7	5	3	6	6	1	6
	15	3	6	5	6	5	5	7	4	6	5
	16	5	4	5	7	7	5	6	3	7	4
	17	6	5	8	5	5	6	6	4	3	3
	18	5	5	5	5	4	4	4	4	6	5
	19	6	5	6	5	4	6	6	6	7	4
	20	5	6	6	7	6	4	7	7	7	7
	21	4	6	6	5	6	5	3	4	4	5
	22	3	5	7	5	7	8	8	8	8	8
	23	2	7	3	5	3	1	6	3	3	7
	24	4	3	6	8	5	4	8	4	6	6
	25	7	7	8	6	8	7	6	7	7	8
	26	4	6	8	7	5	3	7	3	6	6
	27	6	6	2	6	3	5	2	5	2	7

TABLE XVIII

FIRST FIVE EIGENVALUES AND EIGENVECTORS
GROUP A, PROBLEM ONE
EIGENVECTORS

	.198	-.192	.400	.328	-.054
	.273	-.273	.580	-.619	.134
	.268	-.401	.094	.134	-.617
	.303	-.484	-.272	.078	.418
	.299	-.335	-.534	.022	.010
	.329	.339	-.238	-.533	-.342
	.367	.112	-.023	-.065	.151
	.361	.288	.041	.090	-.050
	.362	.247	.129	.402	-.239
	.360	.333	.096	.166	.473
Eigenvalue	9393.43	171.10	88.32	56.96	47.08

TABLE XIX

FIRST FIVE EIGENVALUES AND EIGENVECTORS
GROUP A, PROBLEM TWO

	.257	-.656	.599	-.214	.219
	.291	-.590	-.472	.058	-.537
	.309	-.138	-.424	.472	.671
	.324	.137	-.099	-.260	-.047
	.322	.137	-.217	-.601	.258
	.328	.208	-.064	-.171	-.110
	.331	.120	.310	.276	.035
	.333	.202	.203	.415	-.338
	.331	.195	.003	-.108	-.129
	.329	.165	.201	.099	.015
Eigenvalue	12241.73	76.46	52.48	18.18	9.73

TABLE XX

FIRST FIVE EIGENVALUES AND EIGENVECTORS
GROUP B, PROBLEM ONE
EIGENVECTORS

	.183	-.597	.631	-.172	.243
	.295	.454	.120	.331	.417
	.310	.360	.175	-.398	-.651
	.335	-.125	.146	.759	-.358
	.238	.183	.162	-.154	.185
	.327	.310	.105	-.181	.348
	.342	-.268	-.567	-.208	.120
	.339	-.110	-.395	.106	.082
	.332	-.126	.090	-.119	-.175
	.338	-.251	-.121	-.040	-.091
Eigenvalues	12194.22	81.18	51.50	46.37	31.05

TABLE XXI

FIRST FIVE EIGENVALUES AND EIGENVECTORS
GROUP B, PROBLEM TWO
EIGENVECTORS

	.279	.260	-.157	.478	-.164
	.312	.406	-.166	-.080	.050
	.313	.103	.562	.366	.014
	.334	.370	.097	-.152	-.572
	.328	.022	.212	.178	.629
	.284	-.390	-.243	.360	-.086
	.350	-.127	.310	-.533	.215
	.302	-.224	-.516	.060	.182
	.315	-.613	.171	-.113	-.393
	.337	.158	-.355	-.382	.84
Eigenvalues	8245.09	102.64	70.80	64.49	44.45

TABLE XXII

UNROTATED LOAFINGS FOR COMPONENT CURVES, GROUP A PROBLEM ONE

Subtask	Curve	
	I	II
1	4.003	-.524
2	5.522	-.744
3	5.408	-1.092
4	6.120	-1.320
5	6.039	-.913
6	6.660	.923
7	7.408	.305
8	7.292	.784
9	7.316	.675
10	7.280	.907

TABLE XXIII

UNROTATED LOADINGS FOR COMPONENT CURVES,GROUP A PROBLEM TWO

Subtask	Curve	
	I	II
1	5.922	-1.197
2	6.710	-1.076
3	7.118	-.251
4	7.479	.250
5	7.423	.250
6	7.560	.379
7	7.635	.220
8	7.683	.369
9	7.644	.355
10	7.585	.301

TABLE XXIV

UNROTATED LOADINGS FOR COMPONENT CURVES, GROUP A PROBLEM ONE

Subtask	
1	3.895
2	6.273
3	6.594
4	7.126
5	6.962
6	6.952
7	7.266
8	7.198
9	7.057
10	7.185

TABLE XXV

UNROTATED LOADINGS FOR COMPONENT CURVES, GROUP B PROBLEM TWO

Subtask	
1	4.876
2	5.466
3	5.474
4	5.842
5	5.735
6	4.962
7	6.117
8	5.281
9	5.497
10	5.881

TABLE XXVI

UNROTATED INDIVIDUAL LOADINGS, GROUP A

PROBLEM ONE

Individual	I	II
1	1.182	-.692
2	1.225	-1.003
3	1.102	.048
4	.758	-1.039
5	1.169	-.592
6	.589	-.984
7	1.163	-.504
8	1.028	.062
9	1.225	-1.003
10	1.215	-.932
11	.864	1.496
12	.846	-.294
13	.910	1.997
14	.952	-.253
15	.972	1.304
16	.936	1.707
17	1.030	.913
18	.816	.159
19	.604	-1.989
20	1.061	.418
21	.882	.024
22	1.010	.991
23	1.092	.119

TABLE XXVII

UNROTATED INDIVIDUAL LOADINGS, GROUP A
PROBLEM TWO

Individual	I	II
1	1.044	.118
2	1.094	-.965
3	1.082	-.605
4	1.082	-.605
5	1.071	-.245
6	.701	-.031
7	1.094	-.965
8	1.071	-.245
9	1.038	.835
10	1.035	.763
11	1.082	-.605
12	1.094	-.965
13	1.082	-.605
14	1.038	.835
15	1.071	-.245
16	1.058	-.169
17	.984	1.447
18	.874	1.507
19	.455	-1.844
20	.975	.245
21	1.069	-.529
22	.941	1.752
23	1.071	-.245

APPENDIX F

TABLE XXVIII

VARIANCE-COVARIANCE MATRIX FOR DISCRIMINANT FUNCTION VARIABLE

30.00	-11.00	-8.00	1.33	-7.33	30.33	-128.67	23.33	-25.00
	38.50	-1.00	-2.00	-5.00	10.00	217.50	61.00	28.50
		52.00	-.67	-12.33	58.33	301.33	-117.67	20.00
			31.11	.89	4.44	-68.56	18.11	-4.33
				19.11	-62.44	-103.44	-34.11	-20.66
					837.78	1632.78	670.45	133.67
						8408.27	801.45	642.17
							2229.11	469.67
								290.50

B29824